Prediction of Rheological Properties of Drilling Fluids Using Two Artificial Intelligence Methods: General Regression Neural Network and Fuzzy Logic

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Abstract

The rheological properties of drilling fluids, including viscosity and yield point, are essential for the effectiveness of drilling operations. Inaccurate predictions of these parameters may lead to costly complications during the drilling operation. Among artificial intelligence (AI) methods, the general regression neural network (GRNN) approach and the fuzzy logic method possess high speed of estimation and also less adjustable parameters compared to other methods. Despite the great capability of these two methods, they have seldom been used to predict the rheological properties of drilling fluids. Hence, through programming in MATLAB software, the capabilities of these methods in predicting the rheological properties of drilling fluids were investigated by comparison of their predictions against experimental results. The neural network contained one input layer with three inputs (clay mass, Na₂Co₃ concentration, and Gum Arabic concentration), one hidden layer with 38 neurons, and one output layer with three outputs (apparent viscosity (AV), plastic viscosity (PV), and yield point (YP)). In the fuzzy logic method, the optimal value of the clustering radius was considered 0.1 in this research. Based on the two methods designed, the value of R (about 0.99) and RMSE (about 0.5) between predicted values and the measured values of rheological properties in training and testing data were extremely good. Our findings indicate that both AI methods can be utilized to predict the rheological parameters of drilling fluids with different compositions.

Keywords: Artificial Intelligence, Machine Learning, General Regression Neural Network, Fuzzy logic, Rheological Properties.

1. Introduction

Drilling fluids have multiple tasks in drilling operations, including resisting formation pressure, ensuring wellbore stability, cooling and lubricating the drill bit, cleaning the bottom of the wellbore, and suspending cuttings in the annulus when circulation stops or transporting them to the surface during the drilling process. Therefore, the blood of the drilling operations may be considered as the drilling fluid. Problems or solutions to the problems encountered during drilling operations are directly or indirectly linked to the drilling fluid [1]. Viscosity, yield point, and other rheological properties of drilling fluids are essential parameters for conducting an effective drilling operation. Inaccurately predicting these parameters may lead to expensive drilling problems [2]. When fluid properties are not designed appropriately, various fluid-related problems may occur such as wellbore instability, lost circulation/blowout, and potential formation damage. The oil industry uses sophisticated physics-based methods to anticipate and resolve fluid-related issues. Event detection, hole cleaning modeling, and hydraulics modeling are a few examples. However, these techniques are not always appropriate, and they are computationally costly and challenging to integrate for real-time analysis [3]. Besides, the conventional Bingham plastic and Power law models employed to describe the behavior of non-Newtonian fluids typically have a narrow range of independent variables or have limited application.

Overall, Classical methods in modeling are often laborious, reliant on trial and error, and need iterative adjustment to achieve the best desired outcomes. These models typically need numerous assumptions and simplifications and perform poorly when faced with highly complicated interdependencies [4]. One of the emerging trends in the scientific community that has been incorporated into almost all fields is the use of artificial intelligence (AI). AI is viewed as a tool for comprehending the interactions between complicated structures [1]. Owing to the growing availability

of data and the rapid advancement of AI technology, many machine learning (ML) studies have emerged in various drilling applications, especially in recent years. ML-based methods can be more advantageous than classical analytical or numerical models for a variety of reasons. These reasons include the use of more adaptable model inputs, improved forecasting accuracies, the model capability to facilitate the discovery of new relationships that are not apparent as well as to predict the behavior of systems that are complex with interdependencies between input and output variables, and ultimately, the ability to select the optimal values of model characteristics yielding minimum prediction errors [1, 5].

In terms of data collection, the oil and gas sector is a global leader due to the utilization of data collection devices such as surface and downhole sensors. Massive volumes of information gathered from these sensors are too much usually for the assessment by a human being. ML models, however, make an effort to make certain connections between input and output state variables disregarding the physical dynamic behavior of the system [4]. In fact, ML is an area of AI that focuses on analyzing data, learning from it, and predicting future outcomes. In general, there are several groups of methods and approaches in this regard: supervised learning methods (which comprise regression and classification), unsupervised learning methods (which consist of clustering), semi-supervised learning, and reinforcement learning. However, the most widely utilized technique that maps a set of inputs to the corresponding output(s) is supervised learning [4-5]. Artificial neural network (ANN) with multiple inputs and single/multiple outputs is the most widely used ML technique in drilling mud engineering. ANNs are the most commonly applied AI/ML method in drilling mud engineering accounting for about 50% of the papers published in this area of science. They have capabilities that allow them to resolve difficult and intricate engineering challenges that cannot be solved by classical mathematics or any other traditional ways [1]. In drilling optimization, ANNs can

assist in cases where no clear relationship exists between input and output parameters. Additionally, ANNs can estimate possible outcomes based on a few parameters from the target wells rather than applying the usual industry formulas. Another AI/ML method is fuzzy logic, which is used to deal with non-linear separable datasets. This technique allows us to take into account the degree of truth for several different techniques [4].

The availability of data and the advancements in computation technology have enabled ML methods to gain prominence as versatile in addressing the drawbacks of traditional models for predicting rheological parameters. ANNs and other ML models are employed for the rheological prediction of drilling fluids [2]. Currently, different types of ML techniques are at various levels of integration into drilling fluid engineering, where the most employed are ANNs and the least are case-based reasoning and particle swarm algorithms [1]. There are some recent ML (especially ANN)-related published studies in the literature regarding the prediction of the rheological properties of drilling fluids by AI [see 6-16]. For example, Al-Azani et al. [6] discussed the use of ANNs to develop a model to predict the rheological properties of oil-based drilling fluid. The model was based on 400 data points collected from field measurements, and was found to predict properties accurately with less than 5% error and a correlation coefficient higher than 90%. Elkatatny et al. [7] presented a new approach for determining the rheology parameters of water-based drilling fluid by using ANNs. Rheological properties and flow behavior index were predicted in real-time based on the caliper variables (namely drilling fluid density, Marsh funnel viscosity, and solid percent) that were measured frequently every 15-20 minutes in the well site. The ANN was able to predict the rheological properties with high accuracy. Oguntade et al. [10] discussed the use of ANNs for predicting the properties of water-based mud rheology and filtration. The study used data from laboratory experiments to train the ANN to predict more values

without physical experimentation. The best predictions for rheology properties and filtration properties were obtained by ANNs with 15 neurons and 8 neurons in the hidden layer, respectively. Ismail et al. [12] presented a research paper on the use of grass powder as an environmentally friendly additive to improve the gel strength and viscosity of water-based drilling mud. The study applied ML techniques to the generated rheological data and provided important results in terms of the effectiveness of different particle sizes and weight conditions of the grass additive. An application of ML was presented by Alsabaa et al. [13] to determine the rheological properties of synthetic oil-based mud. ANNs were implemented in developing four models for establishing the rheological characteristics of the synthetic oil-based system. A real-field dataset was utilized for the training and optimization of the proposed models. The predicted rheological properties were statistically acceptable compared to the actual measurements. Al-Obaidi et al. [16] presented a paper regarding the use of ANNs and multiple regression analysis to create new models for real-time prediction of rheological properties of drilling mud. They discussed the importance of mud rheological properties and gel strength in drilling fluid functions. The authors used real field data to create and optimize the ANNs and multiple regression models. The results demonstrated that the ANNs can predict the rheological properties more accurately than multiple regression models. Despite the existence of different ML models in the literature, the general regression neural network (GRNN) approach and the fuzzy logic method are usually preferred due to the high speed of estimation and also less adjustable parameters compared to other AI methods. Published literature indicates that despite the great capability of these two methods, they have seldom been employed to estimate the rheological properties of drilling fluids. Consequently, through programming in MATLAB software, the capabilities of these methods in estimating the rheological properties

of drilling fluids were examined in this study by comparison of their predictions against experimental observations.

2. GRNN and Fuzzy Logic

AI is the science of creating intelligent machines by using computers and through the understanding of human or animal intelligence and finally achieving the mechanism of AI at the level of human intelligence. AI is utilized to solve complex and difficult problems in terms of analytical and logical methods. Comparing AI with human intelligence, humans can observe and analyze issues to make judgments and decisions, while AI is based on rules and procedures already defined in the computer. AI techniques were introduced for those problems that could not be easily solved by functional programming or mathematical methods. The most famous AI branches include [1]: artificial neural networks, support vector machines, fuzzy logic, genetic algorithms, hybrid intelligent systems, particle swarm algorithms, and case-based reasoning.

The ANNs are derived from the biological neural network. Each ANN consists of units called "neurons". Each simple network includes an input layer, a hidden layer, and an output layer. The input layer receives signals from the outside environment (or from other neurons). The hidden layer gathers and processes the input signals and transmits them out through the output layer. Each ANN goes through the stages of training, testing, and implementation. In terms of performance, the ANN has various methods, including the commonly used multi-layer perceptron (MLP) network with back-propagation (BP) algorithm and radial basis function (RBF) neural networks [17-19]. Fig. 1 shows a radial network with R inputs. Radial networks require more neurons than BP networks, but these networks are designed

when the training of BP networks is time-consuming. Besides, these networks have better performance when there are more input vectors [19].



Fig. 1 Neuron model of radial neural network [19]

The input or neuron of this network is different from the input neurons of the BP network. The network input for the radial driving function is the vector distance between the weight vector (w) and the input vector (p), multiplied by the bias (b). In Fig.1, The box ||dist|| takes the input vector (p) and the single-row matrix of the weight (w) and produces a dot product of the two. The driving function of a radial neuron is *radbas*.

GRNN can be considered a normalized radial network that has one hidden neuron for each training unit. This network is a single-pass learning algorithm with a parallel structure that was invented in 1990 and can produce continuous outputs. These networks are based on the probability density function and one of its prominent features is the fast training time and modeling of nonlinear functions. Even with scattered data in a multi-dimensional measurement space, this network provides smooth changes from observational data to other data. The algorithm form of this network can be utilized for any regression problems where there are no assumptions concerning the linearity of the regression. This network does not possess the parameters of the BP network but instead possesses a "smooth factor" whose optimal value is obtained by trial and error [17, 19].

The Fuzzy set theory serves as a valuable tool when in most cases uncertainty or lack of input data related to reservoirs and formations prevail [4, 20]. A fuzzy logic algorithm consists of fuzzy sets formed by the functions of imprecise reasoning and uncertainty. The role of a Fuzzy Logic system is to model the uncertainty that causes the complexity and inaccuracy. The reason behind uncertainties is data insufficiency. Essentially, the output of an event in a random process highly depends on chance or likelihood of occurrence. Hence, probability theory is suitable for handling a problem when uncertainty is a result of event randomness [4]. Classical logic assumes a value of one for true propositions and a value of zero for false propositions, but in "fuzzy logic" there is no need for these values to be zero and one and these propositions are true to some degree. This degree is determined by a function called "membership function" and its range is [0,1]. The goal in fuzzy logic is to relate the input space to the output space by "if-then" rules using Mamdani or Sugeno-type fuzzy inference systems. Fuzzy logic puts the input data into clusters using clustering methods, including the subtraction method, and assigns an output to each cluster, then relates each new input to the aforementioned clusters by considering functions, and based on this and taking into account the mentioned functions, it provides new outputs [21-22].

3. Results and Discussion

In this research, MATLAB software was employed to design GRNN and fuzzy logic models. To train and test these two methods, 48 experimental data on the rheological properties of drilling fluids with different compositions published in Salam et al. [23] were used. Three parameters, namely mass of clay, concentration of Na₂Co₃, and concentration of Gum Arabic, were selected as network inputs, and three

parameters, specifically apparent viscosity (AV), plastic viscosity (PV), and yield point (YP), were chosen as network outputs (Fig. 2). To better recognize the patterns by two methods, all parameters were normalized in the range [-1,1]. Table 1 presents the minimum, maximum, mean, and median values of the network parameters. Table 2 presents the impact of these input parameters on three rheological parameters using SPSS software. According to the Table, all the input parameters have a positive effect on the rheological parameters. The effect of Na₂CO₃ concentration and clay mass on rheological parameters is more noticeable.

Parameter	Minimum	Maximum	Mean	Median
Clay Mass (gr)	22.5	40	35	35
Na ₂ Co ₃ Concentration (%)	0	10	6	7
Gum Arabic Concentration (%)	0	27	10.33	9.5
AV (cp)	1.25	28.83	7.5	6.5
PV (cp)	1	7.83	3.16	3
YP ($lb/100 ft^2$)	0	46.33	6.75	3.5

Table 1 Maximum and minimum values of the data used

Table 2 Correlation matrix between the input and output parameters

		Na ₂ CO ₃	Gum Arabic	Clay Mass	AV	PV	YP
	Na ₂ CO ₃		0.000	0.000	0.433	0.503	0.384
	Gum Arabic	0.000	1	-0.293*	0.222	0.417	0.142
5	Clay Mass	0.000	-0.293*	1	0.555	0.431	0.564
	AV	0.433**	0.222	0.555	1	0.865	0.986
	PV	0.503	0.417	0.431	0.865**	1	0.767
	YP	0.384	0.142	0.564	0.986	0.767	1



Fig. 2 Schematic of the network considered in this study

Out of 48 available data, 38 data were randomly selected for training and 10 data for testing the neural network and fuzzy model. General regression radial neural network (newgrnn in MATLAB) was trained to estimate rheological parameters with different smooth factors. Finally, according to the criteria of correlation coefficient (R) and root mean square error (RMSE) (Eqs. 1-2) for two series of training and testing data, the optimal value of the network smooth factor was considered equal to 0.1. This network considers the number of neurons in the hidden layer as much as the training data (38 neurons).

$$R^{2} = 1 - \frac{\sum_{i=1}^{N} (y_{i} - \hat{y}_{i})^{2}}{\sum_{i=1}^{N} y_{i}^{2} - \frac{\sum_{i=1}^{N} \hat{y}_{i}^{2}}{N}}$$
(1)

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_{i} - \hat{y}_{i})^{2}}$$
(2)

where
$$y_i$$
 denotes the measured value, \hat{y}_i stands for the predicted value, and N indicates the number of data.

The network described above (Fig. 2) was employed to predict the rheological properties of the drilling fluid. As observed in the network designed in the MATLAB software (Fig. 3), this network contains one input layer with three neurons, one middle layer with 38 neurons and a radial activation function, and one output layer with three neurons and a linear activation function.



Fig. 3 Radial structure of GRNN in MATLAB

In MATLAB software, the genfis2 function was used to build the fuzzy model. This function is a Sugeno-type inference system based on the subtractive clustering method. This function constructs a fuzzy system based on subtractive classification by taking an initial classification radius.

In this method, based on the classification radius, the number of categories and so the number of if-then rules or the number of membership functions will be different. The command of the genfis2 function is as follows:

Fismat = genfis2(datain, dataout, r)(3)

Fismat is the name of the created system, *datain* is the matrix of input parameters of the problem, *dataout* is the matrix of output parameters, and r is the classification radius (r is chosen between 0 and 1; the smaller the value of r, the more the number of categories).

The best r is obtained through trial and error, which in our problem was equal to 0.1 according to the values of the correlation coefficient (R) and RMS between the actual

values and the estimated (simulated) values obtained by the fuzzy method in the training and testing data. The type of membership function of inputs is Gaussian and the type of membership function of outputs is linear in this model by default. According to this selected classification radius, the number of rules is equal to 38. To evaluate and simulate the constructed fuzzy system, the *evalfis* command is used. Table 3 presents the results obtained using these two methods for rheological properties.

Model	Training			Testing		
		R^2	RMSE	R	R^2	RMSE
GRNN	AV	1	1.2e-4	0.997	0.994	0.48
	PV	1	5.24e-5	0.95	0.9025	0.55
	YP	1	1.2e-4	0.997	0.994	0.85
Fuzzy	AV	1	4.06e-15	0.997	0.994	0.47
	PV	1	5.15e-15	0.95	0.9025	0.56
	YP	1	6.46e-15	0.997	0.994	0.97

Table 3 The results of the developed predictive models

The predicted values of AV are compared with corresponding actual/experimental observations for training and testing data in Figs. 4-5. The predicted values of PV are compared against corresponding actual/experimental values for training and testing data in Figs. 6-7, and finally, the comparison is shown in Figs. 8-9 for the predicted values of YP and corresponding experimental observations for training and testing data. As observed, there is an excellent agreement between the experimental observations and predicted data in all cases confirmed by R values which are all equal or very close to 1.





ML models have limited applications in terms of interpretability. Classical ML models, while very good in prediction, are usually interpreted as "black boxes" meaning they provide little reasoning for their predicted outputs. Explainable AI (XAI) has been recommended as a potential solution to this challenge to make ML models more interpretable. Despite this, the application of XAI in the field of drilling fluid engineering is still in its preliminary stage, suggesting a substantial opportunity for further development of XAI in this area [2].

4. Conclusion

Two artificial intelligence methods, namely GRNN and fuzzy logic were employed due to their simple structure and high prediction speed to predict the rheological parameters of a specific drilling fluid. The GRNN contained one input layer with three inputs (specifically clay mass, Na₂Co₃ concentration, and Gum Arabic concentration), one hidden layer with 38 neurons, and one output layer with three outputs (AV, PV, and YP). The optimal value of the smooth factor in this network was determined to be 0.1 through trial and error. In the fuzzy logic method, the optimal value of the clustering radius was considered 0.1 in this research. Based on the two methods designed, the values of R (about 0.99) and RMSE (about 0.5) between predicted values and the experimentally measured values of rheological properties in training and testing data were extremely acceptable. In other words, there was an excellent agreement between the experimental observations and predicted data in all cases, confirmed by R values all equal or very close to 1. The results obtained demonstrate that these two methods can be employed to predict the rheological parameters of drilling fluids with different compositions. These parameters can be utilized for the optimal design of drilling hydraulics.

Nomenclature

	Item	Sign	Meaning
	1	Yi	Measured value
	2	Ŷi	Predicted value
-	3	N	Number of data
	4	Fismat	Name of the created system
	5	datain	Matrix of input parameters of the problem
	6	dataout	Matrix of output parameters
	7	r	Classification radius
	8	W	Weight vector
	9	p	Input vector
	10	b	Bias
	11	GRNN	General regression neural network
	12	RMSE	Root mean square error
	13	R	Correlation coefficient

Table 4 List of nomenclatures used in the equations of this paper

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