Identification and Control of MIMO Systems with State Time Delay
(Short Communication)

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Abstract

Time-delay identification is one of the most important parameters in designing controllers. In the cases where the number of inputs and outputs in a system are more than one, this identification is of great concern. In this paper, a novel autocorrelation-based scheme for the state variable time-delay identification for multi-input multi-output (MIMO) system has been presented. This method is based on the stochastic phenomena which are capable of identifying each state variable independent of other variables, a control strategy for controlling such systems; and furthermore confirming the stability criteria. The results demonstrate the effectiveness of the proposed control strategy which has the advantage of confirming the stability, simple implementation and analysis.

Keywords: Time delay, Autocorrelation, State variable delay systems, PI controller, Closed- loop stability

Introduction

In many engineering systems, time-delay in a proportional-integral controller system (i.e., PI) is an integral part of the governing equations. Owing to its intricacy, time-delay identification with either a constant or a variable parameter still poses more exploitation. Actuators, sensors, field networks and wireless communications that are involved in feedback loops usually introducing such delays. In chemical engineering, the measurement of concentration with a gas chromatograph is a typical example where it has a time delay of about 20 minutes. However, in many cases the exact time-delay is either not available or varies with time. The latter, is observed in fouling process in heat exchangers. Hence, the knowledge of time-delay is crucial in designing controller; in particular, model based controllers.

Regarding to the delay knowledge, observers as well as predictors probably constitute the most demanding case of applications. In general, time-delay could be categorized as: delay transfer in input, delay transfer in output or state variable time-delay. Therefore, researchers have presented different schemes for time-delay identification. In their work, time-delay is also referred to as dead time for a system that has time-delay as a hereditary system [1].

In spite of all this progress, a complete or at least satisfactory MIMO generalization of the Lyapunov-based direct adaptive control has not yet been achieved even for the relative degree one case. Indeed, the existing direct MIMO MRAC (Model Reference Adaptive Control) schemes require much more stringent assumptions on the plant than in the SISO ones. The main stumbling block is the high-frequency gain (HFG) matrix Kp. For a MRAC design using the direct adaptation approach, restrictive assumptions on the prior knowledge of Kp have been made. On the other hand, using the indirect adaptation approach requires the estimate of Kp to be nonsingular at all times. Recently, a similar LU factorization was shown to be a key procedure in circumventing the usual restrictive prior assumptions required in direct MRAC design of a 2×2 visual servoing system [2 and 3]. The same visual servoing problem was solved using a factorization of the form Kp = SU, where S and U are symmetric positive definite and U unity upper triangular, respectively [4].

The framework of the standard MRAC control structure is widely used in the control literature for plants without delay; however, two new output feedback adaptive control schemes based on Model Reference Adaptive Control (MRAC) and adaptive laws for updating the controller parameters are developed by Boris and Gutman in 2005 for a class of linear multi-input–multi-output (MIMO) systems with state delay as shown in Figure 1 [5]. In this scheme, they utilized controller rules which have been obtained from matrices of high frequency gain.
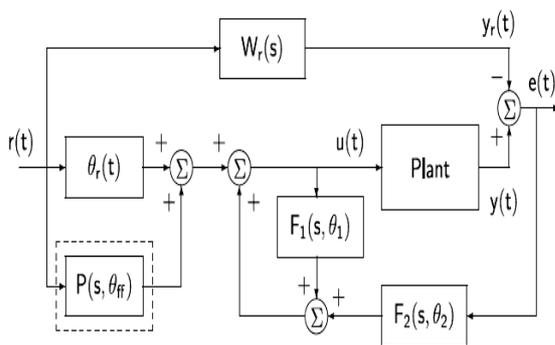
Figure 1: Adaptive control structure with the auxiliary dynamic feedforward P(s, θ_{ff}).

In concrete applications, the delay invariance and delay knowledge remain assumptions coming more from the identification and analysis limits than from technical facts. So, the robustness with regard to the delay estimation (and variation) should receive additional interest. Many physical and chemical systems own multiple input-output, such as distillation column. Controlling such a system constitutes the most demanding case of application. Liu Hsu and co-workers in 2000 implemented a control reference model in controlling such a system and presented controller rules in frequency space [6]. Another method for controlling such systems, is utilizing control loops in parallel. In this technique, each output system is controlled with a controller. The advantage of this algorithm is the simple implementation of the control loop and its practical application.

Time-delay identification scheme adopted in this work

The system that has been utilized in this work has the following form:

$$x(k+1) = A x(k) + \sum_{i=1}^{q} A_i x(k - d_i) + B u(k) \qquad (1)$$

$$x(t=0) = x(0), \quad A_{n \times m}, A_{\tau, n \times m}, B_{n \times m}$$

$$y(k) = c x(k)$$

where B, A_{τ_i} and A are the matrices of equation coefficients which are assumed constant, x state variable, u control variable, n the state number, m the number of system inputs and d_i the amount of delay in each state variable. Analysis of heat exchanger and population ecology are typical examples of such a system [7]. Many researchers have investigated the stability of the above named systems and different algorithms have been presented to control such a system [8]. For example, a robust control of delay systems has been presented using sliding mode control [9]. Furthermore, a model predictive control algorithm has been utilized for the above mentioned system [10].

Prediction of time-delay for the calculation of stability and system control are essential. Furthermore, knowledge of time-delay would cause a cut in computation and make the control algorithm simpler. Diop and co-worker in 2001 utilized the least square method for the identification of time-delay in state variable [11]. They assumed that input to the system at time d was available.

Drakunov and co-workers in 2006 presented a new scheme [12]. In this algorithm, they utilized adoptive method for identification of time-delay.

Therefore, for this algorithm it is necessary to have the values of coefficients for the system. Moreover, according to the Lypunov stability criteria, convergence satisfies when the condition of differential equation for time-delay approaches the actual value. Furthermore, other convergence criteria such as rich input, rich output and the ability of state variable to be differentiable twice, must be satisfied. However, in this

method there is no guarantee for time-delay identification in cases where variables are time dependent. Moreover, no discussion has been made on parameters such as noise and resistivity of the algorithm.

In this work, a new method has been presented for the time-delay identification in state variable for the discretized form of equation1. In this scheme, each state variable could both have independent time-delay and variable time dependent. The advantage of this method is its resistivity to noise which is discussed hereafter.

Time-delay identification by autocorrelation method

A case was considered based on the Drakunov assumptions as demonstrated in Figure 1 [12].

If we could save the x values as a vector, calculation of x (k-d) from the coefficients and the equation is straightforward:

$$x(k-d-1)=A_t^{-1}\begin{pmatrix} x(k+1)- \\ Ax(k)-Bu(k) \end{pmatrix}, k>d \qquad (2)$$

Autocorrelation

In the analysis of stochastic phenomena, the correlation method is widely implemented. The correlation function for two discretized variable are described as follows:

$$C(m)=v(m)*w(m)=\sum_{k=-\infty}^{\infty} v(k)w(k-m) \qquad (3)$$

If the function is continuous, integration instead of addition must be utilized in the above equation. Analysis reveals that if υ and w is two stochastic signals, the above equation is energy signal. Equation 9 could be rewritten as a power signal as follows:

$$P(m)=\lim_{N\to\infty}\frac{1}{2N+1}\sum_{k=-N}^{N} v(k)w(k-m) \qquad (4)$$

In this case, m is the available delay in the υ variable and the autocorrelation (or energy signal) is at highest [13]. Therefore, if in the above correlation we utilize the state

variable vector according to the samples taken in advance, in fact, autocorrelation has been computed and from that time-delay could be easily evaluated. It must be kept in mind that each state variable ought to be analyzed separately (i.e., delay in each state variable is computed independently from other variables).

Control of MIMO systems and the criteria of stability

To analyze the stability, the control for the system was constructed on Figure 2. In this method, PI controller-loops for controlling each output have been adopted. If we consider the equation for PI controller as a one, hence the following relationship would be derived:

$$u_i(k+1)=k_{ci}(e_i(k+1)-e_i(k))+ \\ \frac{k_{ci}}{\tau_i}e_i(k)+u_i(k) \qquad (5)$$

The equations for the control error would be as follows:

$$e_i(k)=y_R(k)-y(k)=y_R(k)- \\ C(Ax(k)+A_\tau x(k-n_\tau)+Bu(k)) \qquad (6)$$

By substituting equation 11 into 12 we would have:

$$u(k+1)= \\ K_c(e(k+1)-e(k))+\Xi e(k)+u(k) \qquad (7)$$

where the above parameters are defined as follows:

$$\Xi=\begin{bmatrix} k_{c1}/\tau_{i1} & 0 & .... & 0 \\ 0 & k_{c2}/\tau_{i2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & k_{cn}/\tau_{in} \end{bmatrix}, \qquad (8)$$

$$K_c=\begin{bmatrix} k_{c1} & 0 & .... & 0 \\ 0 & k_{c2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & k_{cn} \end{bmatrix}$$

After some simplification, controller equation will be reduced to:

$$u(k+1)=-K_cCAx(k)-K_cCA_\tau x(k-n_\tau)-$$
$$K_cCBu(k)+CAx(k-1)+$$
$$CA_\tau x(k-1-n_\tau)+CBu(k-1)+\Xi y_R -$$
$$\Xi CAx(k-1)-\Xi CA_\tau x(k-1-n_\tau)-$$
$$\Xi CBu(k-1)+u(k)$$

(9)

If we combine the system equation with the controller equation, a new system of equation would be obtained:

$$X(k+1)=\Delta_1 X(k)+\Delta_2 X(k-n_\tau)$$
$$+\Delta_3 X(k-1)+\Delta_4 X(k-1-n_\tau)+\Delta_5 y_R$$

(10)

In which the above parameters are defined as follows:

$$X=\begin{bmatrix}x\\u\end{bmatrix},$$

$$\Delta_1=\begin{bmatrix}A & B\\-K_cCA & -K_cCB+I\end{bmatrix},$$

$$\Delta_2=\begin{bmatrix}A_\tau & 0_{n\times n}\\-K_cCA_\tau & 0_{n\times n}\end{bmatrix},$$

$$\Delta_3=\begin{bmatrix}0_{n\times n} & 0_{n\times n}\\CA-\Xi CA_\tau & CB-\Xi CB\end{bmatrix},$$

$$\Delta_4=\begin{bmatrix}0_{n\times n} & 0_{n\times n}\\CA_\tau-\Xi CA_\tau & 0_{n\times n}\end{bmatrix},$$

$$\Delta_5=\begin{bmatrix}0_{n\times n}\\\Xi\end{bmatrix}$$

There are two methods to consider the stability criterion for stability of the closed loop for the above combined system. One is the Lypunov functions and the other is the Δ_i matrices. Since the systems discussed were linear; the latter method was adopted in this work.

Numerical results

To assess the performance of time-delay identification of state variable strategies and control, two systems with two inputs and outputs were considered. Controller parameters for both systems were assumed identical, as are shown in Table 1.

Consider the matrices of data and its parameters for the two case studies considered in this study as depicted in equations 11a and 11b. The simulation results of this study, includes the set point tracking for the first and second exit, exit controller and time-delay identification for the first and second state variables for both case studies which are shown in Figures 3-12.

$$A=\begin{bmatrix}0.3 & 0.1\\0.1 & 0.3\end{bmatrix}, \quad A_\tau=\begin{bmatrix}-0.2 & -0.1\\-0.1 & -0.2\end{bmatrix}$$
$$B=\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}, \quad C=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \quad x0=\begin{bmatrix}-2\\-2\end{bmatrix} \tau=\begin{bmatrix}3\\6\end{bmatrix}$$

(11a)

$$A=\begin{bmatrix}0.2 & 0.2\\0.1 & 0.4\end{bmatrix}, \quad A_\tau=\begin{bmatrix}0.5 & 0.2\\-0.2 & -0.1\end{bmatrix}$$
$$B=\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}, \quad C=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \quad x0=\begin{bmatrix}-2\\-2\end{bmatrix} \tau=\begin{bmatrix}2\\5\end{bmatrix}$$

(11b)

Conclusion

In this paper, a new method for the time-delay identification of systems where there is a delay in the state variable has been proposed. This scheme is based on the stochastic phenomena and is capable of identifying each state variable independent of other variables. This method has fewer limitations in contrast with other methods and does not even needs an initial assumption. Furthermore, in this work a control strategy for controlling such systems with multi input-output has also been presented. Simple implementation, easy analysis and the guaranteed stability are the main advantages of this control strategy.
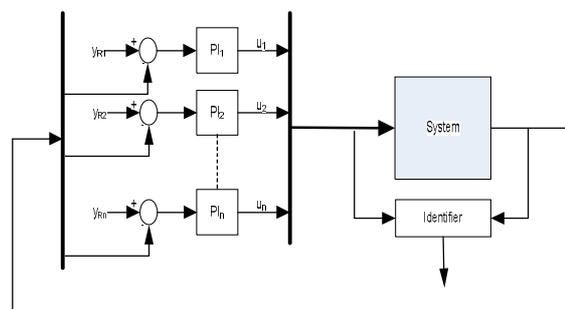
Figure 2: System flow diagram and PI design controller.
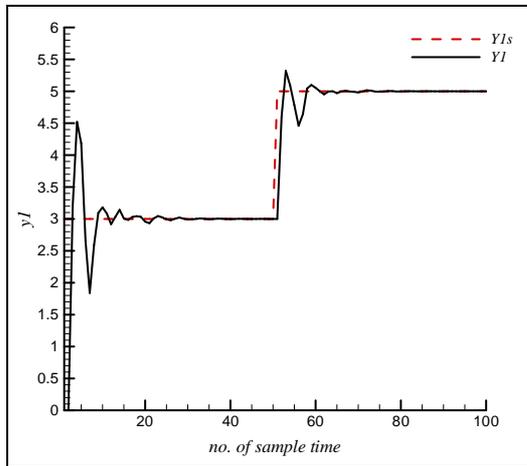
Figure 3: Set point tracking - first exit (case study 1).
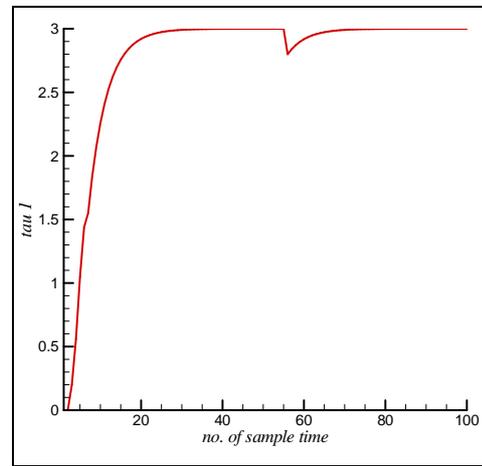
Figure 6: Time-delay identification for the first state variable (case study 1).
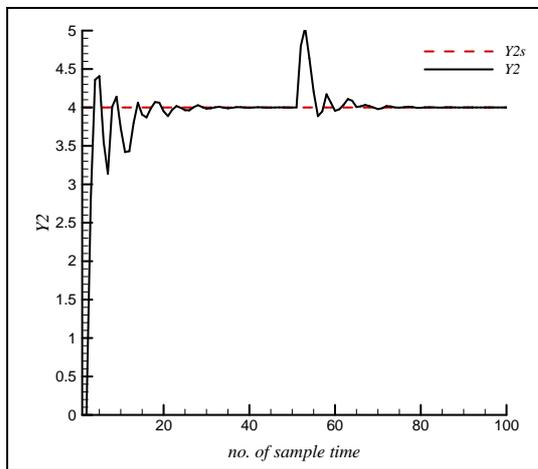
Figure 4: Set point tracking – second exit (case study 1).
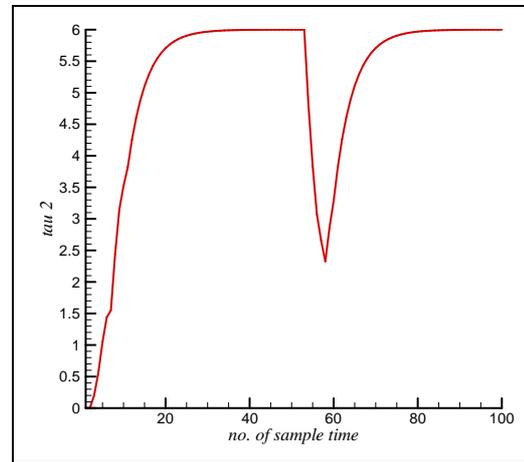
Figure 7: Time-delay identification for the second state variable (case study 1).
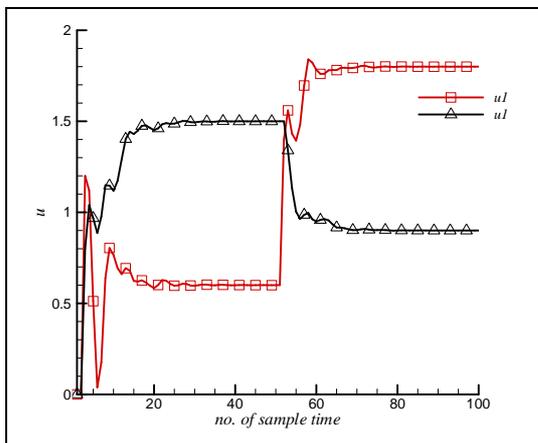
Figure 5: Exit controller (case study 1).
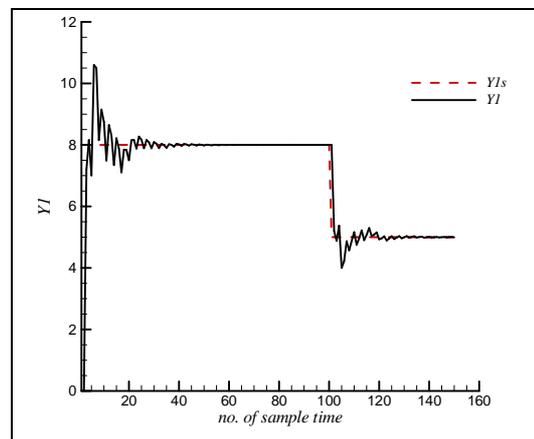
Figure 8: Set point tracking – first exit (case study 2).
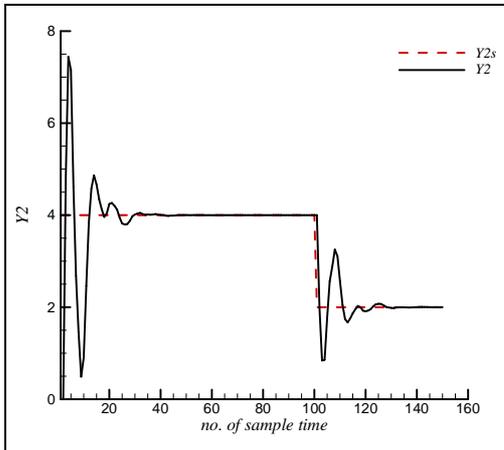
Figure 9: Set point tracking – second exit (case study 2).
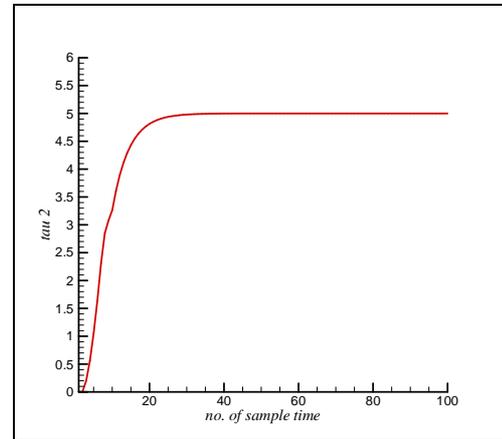
Figure 12: Time-delay identification for the second state variable (case study 2).
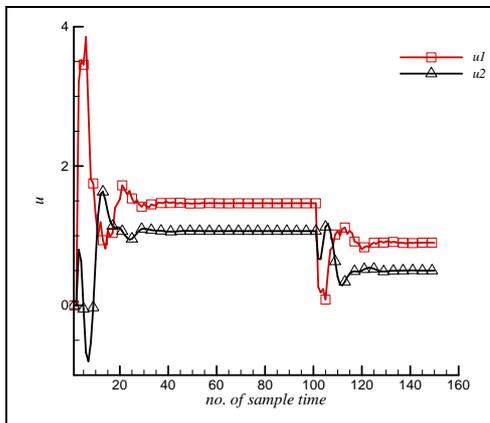
Figure 10: Exit controller (case study 2).

Table 1: PI controller parameters.

τ_I	K_c
1	0.4
1	0.2

Notation

e	Control error
Kc	Parameter in equation 8
p	Identification gain
$P(s, \theta_{ff})$	Special adaptively adjusted dynamic system
r(t)	Reference signal
S	Symmetric positive definite
u	Control variable
U	Unity upper triangular
w	Stochastic signal
τ_I	Integral constant
υ	Stochastic signal
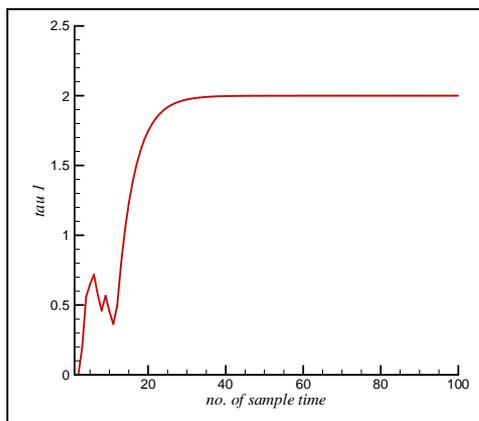
Figure 11: Time-delay identification for the first state variable (case study 2).

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