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RESEARCH PAPER

Sliding Mode Control for Heartbeat Electrocardiogram Tracking Problem

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Abstract

In this paper, we have exploited the first-order sliding mode control method to track the ECG data of the human heart by three different nonlinear control laws. In order to lessen the intrinsic chattering of the classic sliding mode control system, smooth function approximations of the control input, by means of the hyperbolic tangent and the saturation function, were used. The fast Fourier transform was used to evaluate the average chattering frequency of the control inputs. The synthesized control schemes namely SMC-sign, SMC-tanh, and SMC-sat, were able to track the real-world ECG signal with an average root mean square error of 0.0306 and a chattering frequency of 92.7 Hz. The findings show that the sliding mode controllers can be implemented in electronic artificial pacemakers to provide the intended results successfully. Based on today's electronics, the involved frequency range (556.4 Hz for the worst case) is quite acceptable and practical.

Keywords:
Chattering Phenomenon,
Electrocardiogram Signal,
Electronic Pacemaker,
Human Heart,
Nonlinear Control,
Sliding Mode Control

Introduction

The sliding mode control (SMC) is a nonlinear control method for nonlinear dynamic systems that was developed originally in the USSR in the early 1950s. However, it was in the late 1970s when Utkin introduced the method internationally [1]. The SMC has been credited for its robustness and simplicity of implementation, and it is well suited for nonlinear processes subject to external disturbances and substantial model uncertainties. There is an extensive literature on the application of the SMC for chemical engineering process control problems. The interested reader is recommended to study the following references in this regard [2-7].

So far, several mathematical models have been proposed to simulate the operation of the human heart. An electrocardiogram (ECG) signal, being time-varying, reveals useful information about the electrical activity in the cardiac tissue. In other words, a single ECG cycle represents the contraction and relaxation of cardiac muscles that cause the pumping effect of the heart.

In this paper, we have developed a sliding mode control design with control action on the sinoatrial node parameter of a modified Zeeman's heart model to track the ECG data of a normal healthy heart. The control input, being the sinoatrial node parameter, can in real-world be an artificial electronic pacemaker input signal. Thus, as we demonstrate in the sequel, the proposed control design can force an arrhythmic or disordered heart to follow the ECG pattern of a healthy heart. In a study with similar incentive, Thanom and Loh managed to reproduce ECG data of experimental recordings by implementing an observer-based nonlinear feedback control scheme on the second and third-order nonlinear heartbeat models [8]. Also, Sargolzaei et al. [9]

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have adopted an emotional learning control (ELC) strategy, which incorporates time-delayed feedback on the modified Zeeman model. They have shown that their ELC approach was successful at regenerating the ECG data accurately.

Brief Overview of the Sliding Mode Control Method

The SMC has several favorable characteristics such as fast dynamic response and global convergence, ease of implementation, and inherent robustness with respect to process parameters uncertainties and external disturbances. Yet, as one disadvantage, the SMC usually causes a chattering effect in the control responses [10,11].

Without loss of generality, let us consider a nonlinear single input single output (SISO) system in its state-space representation as

$$\begin{cases} \dot{x} = f\left(x,t\right) + g\left(x,t\right)u \\ y = h\left(x,t\right) \end{cases} \tag{1}$$

where u and y are scalar input and output variables, respectively. Also, $x \in R^n$ denotes the state vector. The control aim is to make the system output y track a desired profile y_{des}. For such purpose, the tracking error $e = y - y_{des}$ should remain in a small vicinity of zero after a transient duration.

Put briefly, the SMC synthesis entails two parts: Part 1) The sliding surface design and Part 2) The control input design.

The sliding function, denoted by $\sigma(x);\ Rn \rightarrow R$, is a scalar function of the system states. The associated sliding surface, alternatively known as the sliding manifold, is defined by $\sigma(x)=0$.

It is conventional to define the sliding function in terms of the tracking error and its higher-order temporal derivatives:

$$\sigma = \sigma\left(e, \frac{de}{dt}, \frac{d^2e}{dt^2}, \ldots, \frac{d^k e}{dt^k}\right) \tag{2}$$

A typical linear choice of the sliding function is through the following equation:

$$\sigma = \left(\frac{d\left(\bullet\right)}{dt} + c\right)^k e, \tag{3}$$

where c is an arbitrary positive parameter and $k = r = 1$, with r being the relative degree between the output y and input u.

The characteristic of the sliding surface is such that once the system trajectories cross it, they will be absorbed to the surface and slide along it to the point where $e = 0$.

The second step of SMC design is to devise a control input, which makes the sliding surface attractive. That is, the control law must force the system trajectories to converge to the sliding surface.

The standard, or first-order, sliding mode control law is known as

$$u = -U\,\mathrm{sgn}\left(\sigma\right) = \begin{cases} -U; & \sigma > 0 \\ U; & \sigma < 0 \end{cases} \tag{4}$$

where U is a sufficiently large positive constant. It is obvious that the control input (4) is discontinuous on the sliding surface.

It is worthwhile to mention that the sliding function does not depend on the control input. The conceptualization of the SMC method is illustrated in Fig 1.

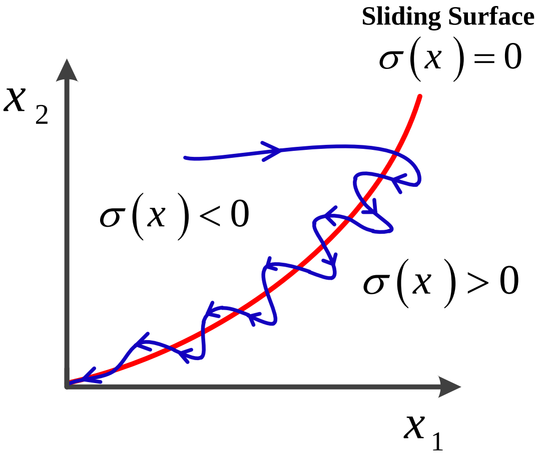

Fig. 1. Illustrative conceptualization of the SMC method. The chattering around the sliding surface can be observed.

The modified Zeeman's model of heartbeat

The original Zeeman's heartbeat model consists of a system of nonlinear ordinary differential equations (ODEs) as follows [12]:

$$\begin{cases} \dot{x}_1 = \dfrac{1}{\varepsilon}\left(x_1^3 - Tx_1 + x_2\right) \\ \dot{x}_2 = x_1 - x_d \end{cases} \tag{5}$$

where x_1 denotes the length of the muscle fiber, x_2 represents the electrochemical activity of the heart and can be measured as the potential across the membrane of the muscle fiber (the ECG signal). Additionally, x_d is the length of the muscle fiber in the diastolic (relaxed) state. The parameter T designates the overall tension in the heart and ε is a time-scale constant.

The modified version of the Zeeman model is given by:

$$\begin{cases} \dot{x}_1 = \dfrac{1}{\varepsilon}\left(x_1^3 - Tx_1 + x_2\right) \\ \dot{x}_2 = x_1 - x_d + \left(x_d - x_s\right)u \end{cases} \tag{6}$$

where x_s denotes the length of the muscle fiber in the systolic (contracted) state. Here, u represents the cardiac pacemaker control signal which directs the heart into the diastolic and the systolic states alternatively [13]. Once appropriately controlled by *u(t)*, the modified Zeeman model can simulate the ECG data of a real heart.

The proposed control design and the results

Our aim is to control the ECG of the human heart (output) by an external pacemaker (control input). Therefore, we obtain the state-space form of the modified Zeeman model as follows:

$$\begin{cases} \dot{x}_1 = \dfrac{1}{\varepsilon}\left(x_1^3 - Tx_1 + x_2\right) \\ \dot{x}_2 = x_1 - x_d + \left(x_d - x_s\right)u \\ y = x_2 \end{cases} \tag{7}$$

In view of Eq. 7, the output-input relative degree is one. Hence, by Eq. 3, we propose the following sliding function:

$$\sigma = \left(\frac{d(\bullet)}{dt} + c\right)^0 e = e \tag{8}$$

where $e = x_2 - x_{2,exp}$ with $x_{2,exp}$ being the real-world ECG data.

We can choose the standard SMC input, readily from Eq. 4, as:

$$u = -U\,\mathrm{sgn}(e) \tag{9}$$

The numerical values of the model parameters used in our case-study are listed in Table 1 [14].

Table 1. Numerical values of the modified Zeeman model parameters

Parameter	Value
ε	0.2
T	1
x_d	1.0240
x_s	-1.3804

The initial conditions for the system (7) were taken as $[x_1(0), x_2(0)] = [-0.3, 0.01]$. The real-world ECG data was taken from PhysioBank ECG databases (https://physionet.org/data/#ecg).

The simulation results of the controlled system are presented in Fig. 2. As can be seen, the SMC-sign design perfectly tracks the non-smooth, and with numerous kinks, the behavior of the ECG signal.

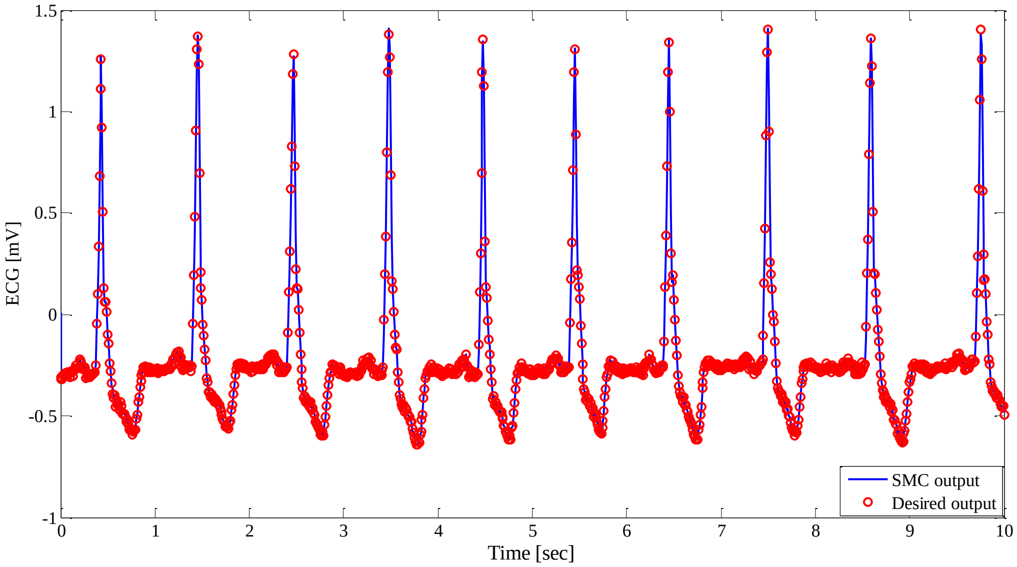

Fig. 2. Control output of the SMC-sign design (U = 20) in comparison with the experimental ECG data.

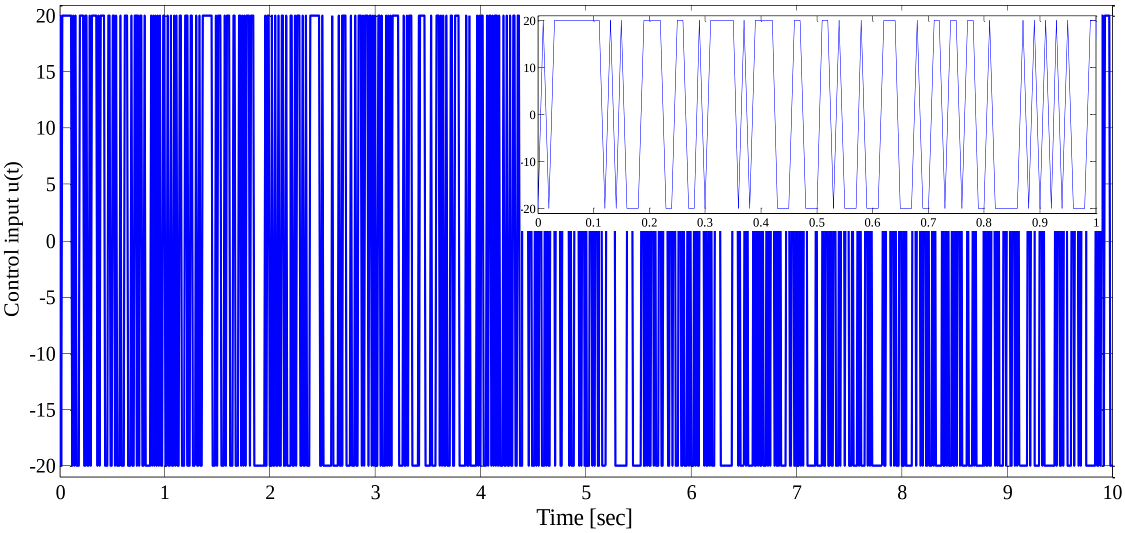

Fig. 3. The control input for the SMC-sign design (U = 20).

The chattering effect can clearly be observed in the control input from Fig. 3.

In order to overcome the chattering phenomenon, it has been suggested to change the discontinuous sign function control law with a corresponding smooth approximation with the help of hyperbolic tangent function:

$$u = -U \tanh\left(\frac{\sigma}{h}\right) \tag{9}$$

where h is a small auxiliary positive constant.

Similarly, the use of the saturation function has been shown to be helpful in decreasing the involved chatterings.

$$u = -U \ \mathrm{sat}(\sigma, h) = -U \ \frac{\sigma}{|\sigma| + h} \tag{10}$$

Fig. 4 depicts the model output of the SMC-tanh control system compared with the target ECG data. Apparently, the designed control system has been almost successful at tracking the real-world heartbeat data. Also, the adopted control input for this design is shown in Fig. 5. In view of Figs. 3 and 5, we can visually recognize that the chattering in the SMC-tanh design is lowered.

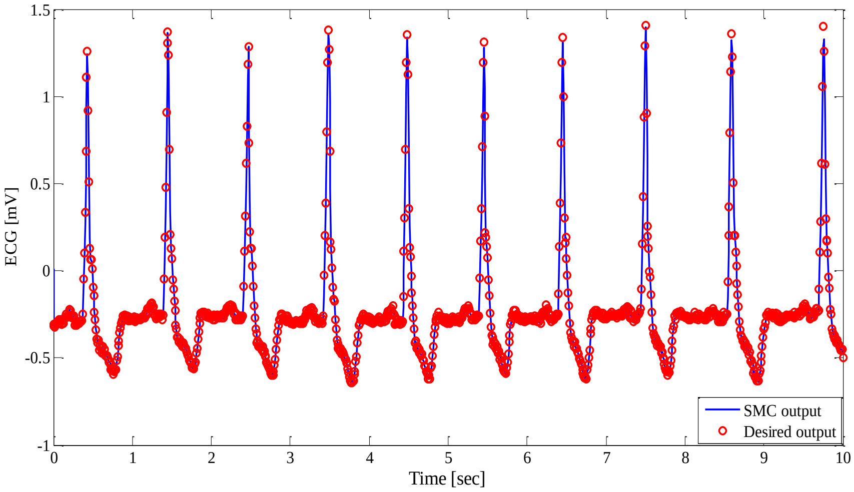

Fig. 4. Control output of the SMC-tanh design (U = 20, h = 0.1) in comparison with the experimental ECG data.

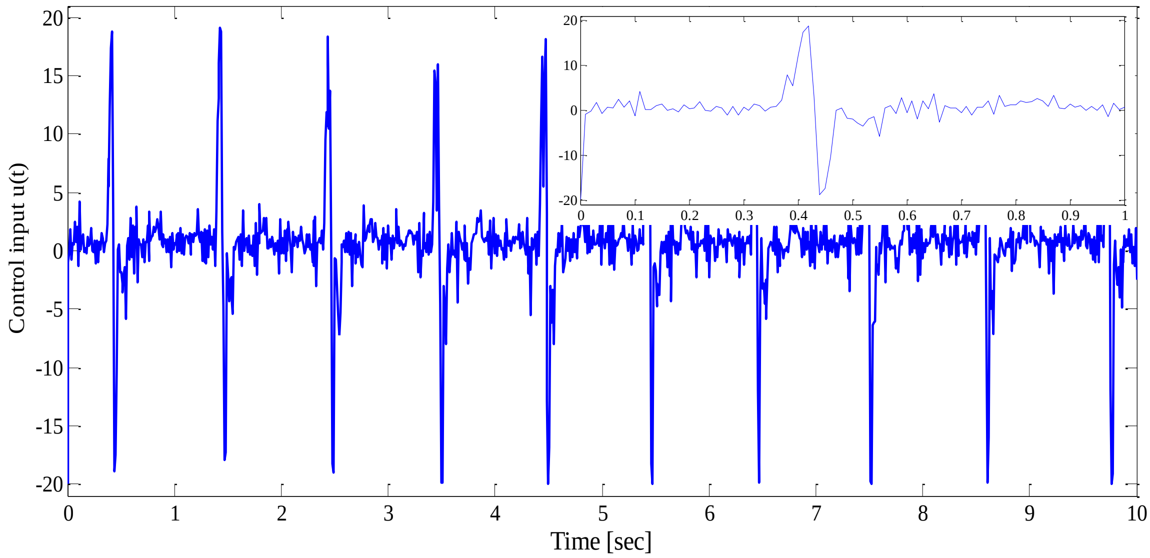

Fig. 5. The control input for the SMC-tanh design (*U* = 20 and *h* = 0.1).

The performance of the SMC-sat control scheme is presented in Fig. 6. We can identify some mismatches between the system output and the desired values. However, according to Fig. 7, the relevant control input is less chattering compared with the SMC-sign design.

For error analysis of the proposed control designs, the root mean square error (RMSE) of the control system output and the reference data were used. The computational formula for such error is as follows:

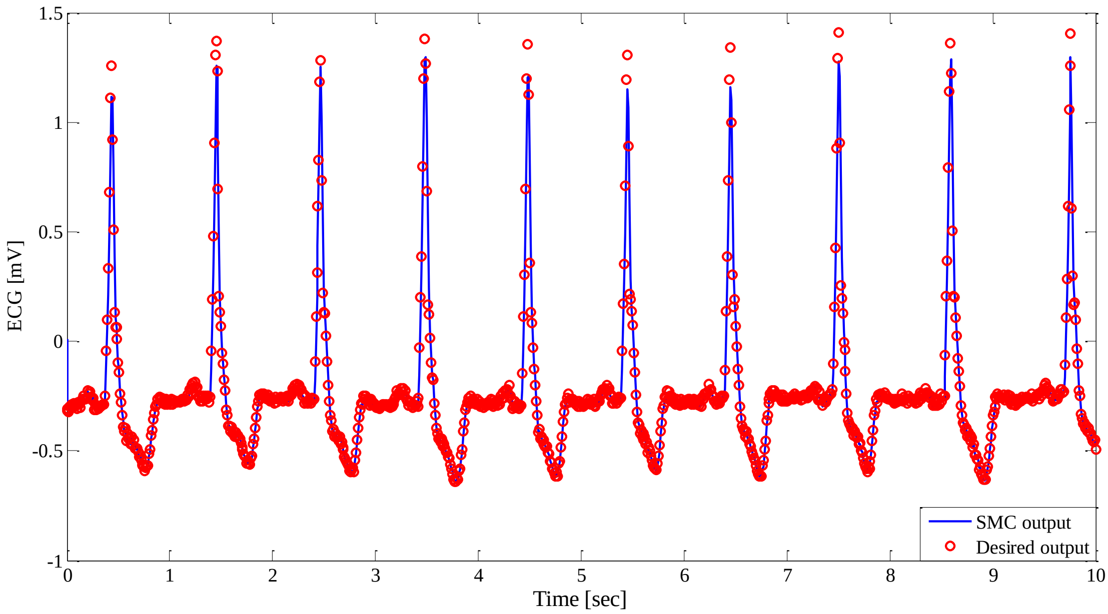

Fig. 6. Control output of the SMC-sat design (*U* = 20 and *h* = 0.1) in comparison with the experimental ECG data.

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i^{\text{ctl. sys.}} - y_i^{\text{ref.}}\right)^2}{n}} \tag{11}$$

In order to evaluate the chattering phenomenon quantitatively, we computed the average oscillation frequency from the fast Fourier transform (FFT) of the control input signal and name it as the chattering frequency.

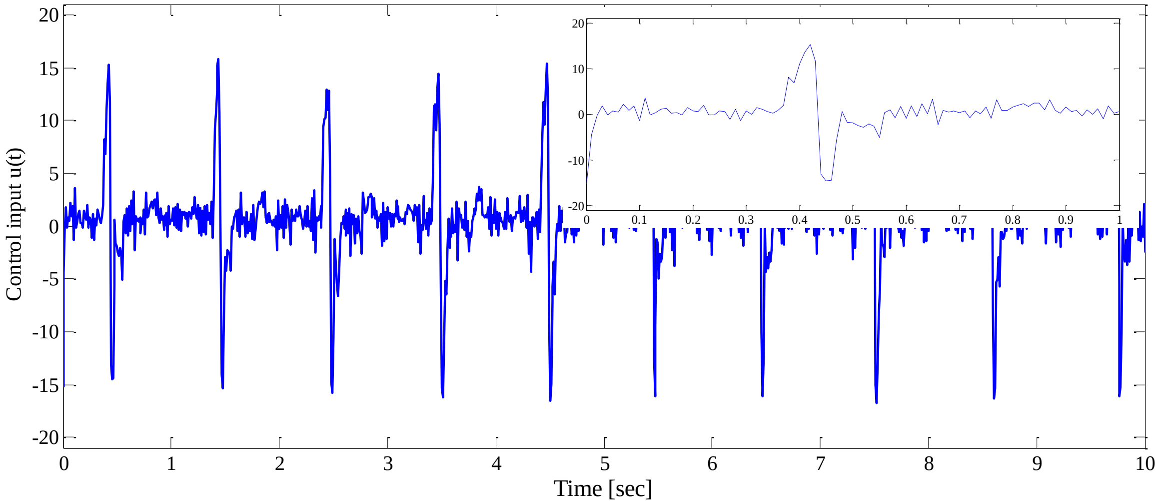

Fig. 7. The control input for the SMC-sat design (U = 20 and h = 0.1).

Also, for selecting an optimal control system, a performance index was devised by the following equation:

$$\text{Performance Index} = \frac{100}{0.4 \times \text{Normalized Chattering Frequency} + 0.6 \times \text{Tracking Error}} \quad (12)$$

where the normalized chattering frequency is the chattering frequency divided by the experimental heartbeat frequency (0.98039 Hz for our study).

In formulating the performance index, we postulated that a better control scheme is the one with lesser chattering in the control response and the one which produces a lower tracking error. We considered weight factors of 40 and 60% for these two aspects, respectively. The reason behind such a decision was that chattering is most troublesome once the final control element (FCE) actuates mechanically. Here, out FCE is an electronic pacemaker with integrated circuits that readily handle high-frequency dynamics (~ a few thousand Hertz). Hence, a smaller importance factor was dedicated to the chattering in the performance index.

According to Table 2, SMC-sign control system is the least favorable. This is due to the substantial chattering in the controller output. On the other hand, the SMC-tanh system can be considered as the most practical one. The SMC-sat scheme is relatively acceptable in terms of the chattering effects but it is faulty when producing the ECG data.

Table 2. Performance assessment of the proposed control laws.

Control Scheme	Tracking Error (RMSE)	Chattering Frequency (Hz)	Performance Index
SMC-sign	0.0306	556.4	0.44
SMC-tanh	0.0448	106.6	2.2978
SMC-sat	0.0701	92.7	2.6410

Still, as discussed above, only as long as there are no mechanical pieces involved in the FCE, i.e. the pacemaker, we can reassure that the SMC-sign control design is highly suitable.

Conclusions

The standard sliding mode control method was applied to track the experimental ECG data. In addition, the smooth function approximation, i.e. hyperbolic tangent and saturation functions, was employed to attenuate the inherent chattering of the original SMC scheme. All three synthesized control designs managed to achieve the preset goal. Nevertheless, the SMC scheme with the saturation function control law was evaluated optimum based on a performance index defined so that to incorporate the effects of the tracking error and the chattering frequency.

References

[1] Izosimov DB, Matic B, Utkin VI, Sabanovic A. Using sliding modes in control of electrical drives. InDokl. ANSSSR 1978 (Vol. 241, No. 4, pp. 769-772).

[2] Xiao T, Li HX. Sliding mode control design for a rapid thermal processing system. Chemical Engineering Science. 2016 Apr 2;143:76-85.

[3] Camacho O, Smith CA. Sliding mode control: an approach to regulate nonlinear chemical processes. ISA transactions. 2000 Apr 1;39(2):205-18.

[4] Dimassi H, Winkin JJ, Wouwer AV. A sliding mode observer for a linear reaction–convection–diffusion equation with disturbances. Systems & Control Letters. 2019 Feb 1;124:40-8.

[5] Gonzalez J, Fernandez G, Aguilar R, Barron M, Alvarez-Ramirez J. Sliding mode observer-based control for a class of bioreactors. Chemical Engineering Journal. 2001 Apr 15;83(1):25-32.

[6] Chen CT, Peng ST. Design of a sliding mode control system for chemical processes. Journal of Process Control. 2005 Aug 1;15(5):515-30.

[7] Chen CT, Peng ST. A sliding mode control scheme for non-minimum phase non-linear uncertain input-delay chemical processes. Journal of Process Control. 2006 Jan 1;16(1):37-51.

[8] Thanom W, Loh RN. Observer-based nonlinear feedback controls for heartbeat ECG tracking systems. Intelligent Control and Automation. 2012 Aug 1;3(03):251.

[9] Sargolzaei A, Yen KK, Abdelghani MN. Control of nonlinear heartbeat models under time-delay-switched feedback using emotional learning control. International Journal on Recent Trends in Engineering & Technology. 2014 Jan 1;10(2):85.

[10] Utkin V, Guldner J, Shi J. Sliding mode control in electro-mechanical systems. CRC press; 2009 May 1.

[11] Perruquetti W, Barbot JP. Sliding mode control in engineering. CRC press; 2002 Jan 29.

[12] Zeeman EC. Differential equations for the heart beat and nerve impulse, in “Towards a Theoretical Biology,” Vol. 4. Edinburgh Univ. Press, Edingburgh. 1972:8-67.

[13] Jones DS, Plank M, Sleeman BD. Differential equations and mathematical biology. Chapman and Hall/CRC; 2009 Nov 9.

[14] Priyadarshi PA, Kannaiyan S. Estimation of Human Heart Activity Using Ensemble Kalman Filter. Sensors & Transducers. 2017 Feb 1;209(2):90.