Thermodynamic Modeling of the Gas-Antisolvent (GAS) Process for Precipitation of Finasteride

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Abstract
Experimental study of the effect of gas antisolvent (GAS) system conditions on the particle size distribution of finasteride (FNS) requires a thermodynamic model for the volume expansion process. In this study, the phase behavior of the binary system including carbon dioxide and Dimethyl sulfoxide, and a ternary system comprising carbon dioxide, dimethyl sulfoxide, and Finasteride was studied. The Peng-Robinson equation of state was employed for the evaluation of the fluid phases and a fugacity expression to represent the solid phase. By developing an accurate predictive model, the GAS operating conditions can be optimized to produce particles with no need for a large number of experiments. First, the critical properties of the FNS were evaluated by the group contribution methods. The method of Marrero and Gani was also selected to predict the normal boiling point, critical temperature, and critical pressure. The correlation of Edmister was chosen for the prediction of the acentric factor. The lowest pressures for the ternary system at 308, 318, 328, and 338 K were 7.49, 8.13, 8.51, and 9.03 MPa, respectively. The precipitation of the dissolved finasteride happened at these operating pressures.

Keywords:
Finasteride, Genetic Algorithm, Group Contribution, Supercritical Fluid, Thermodynamic Modeling

Introduction

The processes of preparing pharmaceutical products require the generation of very small and pure drug particles. The use of small particles in the drug delivery applications will decline the number of required particles [1-4]. Recently, supercritical fluids (in particular, carbon dioxide) have been used for processing pharmaceutical particles through different techniques, such as rapid expansion of supercritical solutions (RESS), gas antisolvent precipitation (GAS), solution enhanced dispersion by SCF (SEDS), supercritical antisolvent methods (SAS), and aerosol supercritical extraction system (ASES) [5-8]. Supercritical carbon dioxide (SC-CO₂) could act as either a solvent or an anti-solvent. SC-CO₂ has been studied as a suitable alternative to conventional solvents that may damage sensitive compounds like pharmaceuticals. It has low critical temperature and pressure (Tc = 304.2 K and Pc = 7.38 MPa, respectively). SC-CO₂ offers additional benefits due to its nontoxicity, cost-effectiveness, nonflammability, and environmental-friendliness [5,9-12]. The gas anti-solvent (GAS) system can resolve the problem of the low solubility of most substances, especially pharmaceuticals, in supercritical fluids. The gas anti-solvent process has three components: i) a low-volatile organic solvent, ii) a highly volatile anti-solvent, and iii) a solute. The solute and anti-solvent must have a definite level of solubility in the solvent. In the GAS process, a solid dissolved in an organic solvent and a high-pressure gas (in particular, carbon dioxide (CO₂) at a given temperature and pressure) are injected into the liquid-phase solution [13-17]. Submicron particle sizes have been obtained using the GAS process from various drugs such as mefenamic acid-nicotinamide.

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(MEF-NIC) cocystal [18], paracetamol into silica aerogel [19], mefenamic acid (MEF) and polyvinylpyrrolidone (PVP) [20], ibuprofen with (R)-phenylethylamine [21] and 5-Fluorouracil [22]. In this process, the solubility of carbon dioxide as an anti-solvent gas is very high in the liquid solvent and may cause a volume expansion in the liquid solvent. Therefore, the solute solubility in the expanded liquid phase reduces by carbon dioxide. As a result, the precipitation of the dissolved compound will occur giving a rise to small particles with a fine size distribution.

In the gas anti-solvent system, process variables, such as temperature, pressure, and solute initial concentration can dramatically influence the morphology of the particles, as well as their particle size and particle size distribution [5,16,22-28].

For evaluation of the appropriate operating conditions for such a process and optimizing the effective parameters, it is necessary to know the thermodynamic model of the volume expansion in the GAS process before performing any experiments [29,30].

The most accurate EoS (presented by De la Fuente Badila et al. [27]) was employed to estimate the relative molar volume variations. For example, Esfandiari et al. [31] optimized for a binary (DMSO-CO$_2$) and a ternary (DMSO–CO$_2$-ampicillin) system. The thermodynamic modeling was conducted based on the Peng-Robinson equation of state with a linear combination of Vidal and Michelsen mixing rules (PR-LCVM). The optimal condition for ampicillin precipitation was determined through modeling the volume expansion and phase equilibrium. Optimum conditions were investigated for binary systems of CO$_2$-diethyl succinate and CO$_2$-ethyl acetate as well as the ternary systems of CO$_2$-diethyl succinate-ethanol and CO$_2$-diethyl succinate-ethyl acetate [32].

Moreover, Sue et al. [33] employed volume-translated Peng-Robinson (VTTR) to assess the total volume expansion of DMSO and CO$_2$. To this end, the relative molar volume variation was evaluated in terms of pressure using VTTR-EoS. The minimum pressure at 308.15 K was determined to be 7.65 MPa.

Finasteride is a 5α-reductase inhibitor. It is specifically a selective inhibitor of type II and III isoforms of the enzyme. It has been applied for the treatment of benign prostatic hyperplasia (BPH) symptoms in men with prostate enlargement, prostate cancer, and androgenetic alopecia. This drug was used to stimulate hair growth in men with mild to moderate androgenic alopecia (male pattern alopecia, hereditary alopecia, common male baldness) [34,35].

Following biopharmaceutical classification, finasteride (FNS) is a member of class 2 drugs with high permeability and poor solubility in water. Its water solubility was reported to be 0.05 mg/ml at the pH range of 1 to 13 which is very low. It is a weakly acidic drug with pKa of 15.9 [34]. FNS is also poorly soluble in supercritical carbon dioxide (molar fraction solubility $10^{-5}$ to $10^{-4}$ at 308 ≤ T ≤ 348 K and 121 ≤ P ≤ 355 bar) [36].

This work is aimed to study the phase behavior of binary (DMSO-CO$_2$) and ternary (CO$_2$ +DSMO+Finasteride) systems. The Peng-Robinson equation of state (EoS) with vdW2 mixing rules was also used to represent the fluid phases and the fugacity of the precipitated solid phase. The volume expansion and phase equilibrium were also modeled to optimize the condition for finasteride precipitation.

**Thermodynamic Framework**

Thermodynamic studies in gas-liquid systems are often difficult for products that are distributed between the liquid and the vapor phase. Therefore, the description of the phase equilibrium is usually performed by an equation of state (EoS). Usually, cubic equations of state are used to develop the methods for estimating the vapor-liquid thermodynamic equilibrium. Furthermore, the equation of states should be modified to evaluate the fractions of the liquid and vapor phases in the mixture of hydrocarbons. In the GAS systems, high-pressure CO$_2$ is injected into the
solution, which will cause a volume expansion in the solution giving rise to the particle precipitation in a short period. In this regard, the optimal conditions of finasteride were calculated for binary (CO$_2$-DMSO) and ternary (DMSO-CO$_2$-finasteride) systems. For this purpose, VLE data were modeled via the Peng-Robinson equation of state with van Der Waals Mixed Rules (vdW2) before the experiments. In the modeling of the GAS process, the pressure and temperature of all phases are assumed equal. Additionally, due to the low volume of precipitator and mixing of liquid and gas phases, mass transfer resistance is not considered [27,31,37,38].

**Binary System Anti-Solvent (1)-Solvent (2)**

As the classical liquid-phase volume expansion is given for the determination of a proper solvent and operation conditions, De la Fuente Badilla et al. [39] represented that the use of the relative molar volume changes of the liquid phase is more convenient for optimization of the GAS process condition. Therefore, the following definition can be presented for the relative molar volume change [31,37,39,40]:

\[
\frac{\Delta \bar{V}}{\bar{V}} = \frac{V_L(T, P) - V_2(T, P_0)}{V_2(T, P_0)}
\]  

Eq. 1 only represents the relationship between molar volumes of the liquid phase in the mixture and the pure solvent. In Eq. 1, $V_2(T, P_0)$ demonstrates the molar volume of a pure solvent at the system temperature and reference pressure (usually at 101.325 KPa), while $V_L(T, P, X_1)$ stands for the molar volume of the liquid phase at a given temperature and the pressure of the binary mixture, and $X_1$ represents the mole fraction of CO$_2$ dissolved in the liquid phase.

**Ternary Systems Anti-Solvent (1)-Solvent (2)-Solute (3)**

In this research, the methods presented by De la Badilla et al. [39], and Shariati and Peters [37] were proposed for modeling. The equilibrium criteria for the solid-liquid-vapor three-phase equilibria implies equal temperature, pressure, and fugacity of components (CO$_2$, DMSO, and finasteride) in the three possible phases. Therefore, the equilibrium criteria for the anti-solvent (1)-solvent (2)-solute (3) can be written as:

\[
\frac{\hat{\phi}_1^L}{\hat{\phi}_1^V} x_1 - y_1 = 0
\]  

\[
\frac{\hat{\phi}_2^L}{\hat{\phi}_2^V} x_2 y_2 = 0
\]  

\[
\frac{\hat{\phi}_3^L}{\hat{\phi}_3^V} x_3 y_3 = 0
\]  

\[
k_i = \frac{y_i}{x_i} = \frac{\hat{\phi}_i^L}{\hat{\phi}_i^V}
\]

Eqs. 2 to 4 represent the equilibrium condition for the three phases of liquid, and gas in the GAS process. Eqs. 2 and 3 express the liquid-vapor equilibrium conditions for the two-phase system, while the liquid-vapor equilibrium conditions of the three-phase system are presented in Eqs. 2 to 4. Some assumptions for solid-liquid equilibrium needed during the calculations are [28,36]:

1. solubility of solvent in the solid phase was negligible,
2. solubility of anti-solvent in the solid phase was negligible, and
3. the solid phase was the pure solute.
Using this assumption, Eq. 6 can be derived.

\[
\frac{\Phi^s_3}{\Phi^L_3} x_3 = 0
\] (6)

In Eq. 6, \( \Phi^s_3 \) demonstrates the solute fugacity coefficient in the solid phase. The next limitations were applied for liquid and vapor phases [39,40].

\[
\sum_{i=1}^{3} x_i - 1 = 0
\] (7)

\[
\sum_{i=1}^{3} y_i - 1 = 0
\] (8)

Eqs. 2 to 7 indicate a set of six equations with six unknown components in the fluid phases in a certain temperature and pressure. For the description of the fluid phases, the Peng-Robinson equation of state is expressed by [37,41]:

\[
P = \frac{R T}{v-b} \frac{a(T)}{v(v+b)+b(v-b)}
\] (9)

where \( v \) is the molar volume. The quadratic mixing rules in mole fraction for \( a \) and \( b \) are used as follows:

\[
a = \sum_{j} \sum_{i} x_i x_j a_{ij}
\] (10)

\[
b = \sum_{j} \sum_{i} x_i x_j b_{ij}
\] (11)

where \( j \) and \( b_{ij} \) are the cross energetic parameter and the cross-co-volume parameter, respectively. \( a_{ij} \) and \( b_{ij} \) are calculated as follows:

\[
a_{ij} = (a_i a_j)^{0.5} (1-k_{ij})
\] (12)

\[
b_{ij} = \left( b_i + b_j \right)^2 (1-l_{ij})
\] (13)

In Eqs. 12 and 13, \( k_{ij} \) and \( l_{ij} \) are the interaction parameters; while \( a_i \) and \( b_i \) can be given by the following equations:

\[
a = 0.45724 (R^2 T_c^2) \frac{p_c}{P_c} \alpha(T)
\] (14)

\[
b = 0.0778 \frac{R T_c}{p_c}
\] (15)

The temperature-dependent energetic parameter, \( \alpha(T) \), can be given by the following equations:

\[
\alpha^{1/2} = 1 + k(1-T_r^{1/2})
\] (16)

\[
k = 0.37464 + 1.54226 \omega - 0.26992 \omega^2
\] (17)

where \( k \) and \( \omega \) are the pure compound parameters of the component \( i \), and the acentric factor of the solid compound, respectively. In this study, the phase equilibrium was modeled by the Peng-Robinson equation of state (PR-EoS) with vdW2 mixing rules. By applying thermodynamic manipulations, analytical equations can be obtained from the fugacity of the fluid phases. As the Peng-Robinson EoS fails to show the behaviour of the solid phase, another definition has to be applied for the fugacity of the solid solute [31,37]. The solid phase fugacity coefficient can be defined by the following equation [40]:

\[
ln \Phi^s_3 = ln \Phi^L_3 + \frac{\Delta H_T}{R} \left( \frac{1}{T_{tp}} - \frac{1}{T} \right) + \frac{\nu_T}{RT} (P - P_{tp})
\] (18)
where, \( \ln \varphi^L_3 \) shows the fugacity coefficient of the pure solute in the sub-cooled liquid phase at the temperature \( T \), and pressure \( P \). \( T_{tp}, P_{tp}, v_{tp}, \) and \( \Delta H_{tp} \) are triple point temperature, triple point pressure, solute molar volume at the triple point, and heat of fusion at the triple point, respectively. These parameters are necessary for the calculation of the fugacity coefficient of the pure solid phase. The physical properties of finasteride used for the solid phase fugacity evaluation were also calculated. In the present study, the method of Marrero and Gani was proposed for calculating the finasteride properties.

### Genetic Algorithm

The genetic algorithm (GA) is a meta-heuristic method inspired by the efficiency of natural selection in biological evolution. The genetic algorithm has shown promising potential in various applications. It demonstrates a powerful problem-solving ability that can be successfully applied to a wide variety of complex combinatorial problems. GAs are global search methods made of a few principles like selection, crossover, and mutation. Briefly, GA involves a randomly-generated initial population, a fitness function, and the development of new generations via the application of genetic operators, namely selection, crossover, and mutation \([42-44]\).

In the present work, an effective algorithm was proposed based on a genetic algorithm (GA) technology to efficiently estimate the adjustable parameter of the thermodynamic model. Moreover, finding the global optimum with high probability and significance is not sensitive to the initial estimates of the unknown parameters and the tuning parameters of the model. It was used to select the optimal binary interactive parameters of the PR model. The binary interaction parameters \( k_{ij} \) and \( l_{ij} \), present in Eqs. 12 and 13, have been optimized using \( |\Delta k_i| < \varepsilon \) in this work for each temperature \([31]\).

### Results and discussion

The van der Waals mixing rules with two parameters (vdW2) was applied in this work for mixture calculations. The fluid phase behavior of the binary and ternary systems (DMSO-CO\(_2\) and DMSO-CO\(_2\)-finasteride) was predicted by the Peng-Robinson equation of state. The first step in the calculation of the phase equilibrium data using the PR EoS is the estimation of boiling point, critical properties, and acentric factor of the drug compounds which can’t be measured experimentally. For heavier hydrocarbons, the critical thermodynamic properties are not available at all. Therefore, empirical methods such as group contribution methods or molecular level simulation are often used to evaluate these critical parameters \([45,46]\). In this research, a group contribution method proposed by Marrero and Gani \([47]\) (considering 182 functional groups) was used to estimate the finasteride properties. In this method, the structure of the compound is determined and the molecules of a compound are collected from different groups. These groups are first-order groups, second-order groups, and third-order groups. The distribution and population of each group are determined. The first-order groups are described as a wide variety of organic compounds. The second and third-order groups are used to describe the molecular structure of compounds. Therefore, the property calculation is performed at three levels. The initial approximation is determined by the first level. Then, the second and third levels refined the initial approximation. The function of property-estimation is given by the following equation:

\[
 f(X) = \sum_i N_i C_i + w \sum_j M_j D_j + z \sum_k O_k E_k
\]

(19)
where $C_i$ is the contribution of the first-order group of type-i that happens $N_i$ times, $D_j$ is the contribution of the second-order group of type-j that happens $M_j$ times, and $E_k$ is the contribution of the third-order group of type-k that occurs $Q_k$ times in a compound [48]. The correlation of Edmister was chosen for the prediction of the acentric factor. The critical properties and the acentric factors of FNS, DMSO, and CO$_2$ are listed in Table 1. Furthermore, the physical properties of FNS needed in Eq. 18 are listed in Table 2.

### Table 1. Critical properties and acentric factor of substances

<table>
<thead>
<tr>
<th>Substance</th>
<th>$T_c$ (K)</th>
<th>$P_c$ (bar)</th>
<th>$\omega$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO$_2$</td>
<td>304.13</td>
<td>73.8</td>
<td>0.224</td>
<td>[46]</td>
</tr>
<tr>
<td>DMSO</td>
<td>706.9</td>
<td>58.5</td>
<td>0.45</td>
<td>[46]</td>
</tr>
<tr>
<td>FNS</td>
<td>902.22</td>
<td>16</td>
<td>0.42</td>
<td>This work</td>
</tr>
</tbody>
</table>

### Table 2. The physical properties of FNS required in Eq. 17

<table>
<thead>
<tr>
<th>Substance</th>
<th>$T_{lp}$ (K)</th>
<th>$P_{lp}$ (pa)</th>
<th>$v_{lp}$ (cm$^3$ mol$^{-1}$)</th>
<th>$\Delta H_{lp}$ (kJ mol$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FNS</td>
<td>530.85</td>
<td>8E+3</td>
<td>397.7</td>
<td>165.84</td>
</tr>
</tbody>
</table>

### Binary System

By defining the molar volume of the liquid solution, the best combination of DMSO and CO$_2$ takes place when a minimum pressure occurs in the volume expansion curve [49,50].

$$v = x_1 \bar{v}_1 + x_2 \bar{v}_2$$  \hspace{1cm} (20)

In Eq. 20, $v$ is the molar volume of solution, $x_i$ shows the mole fraction of CO$_2$ in the liquid phase, $\bar{v}_1$ represents the partial molar volume of CO$_2$, $x_2$ denotes the mole fraction of the solvent in the liquid phase, and $\bar{v}_2$ stands for the partial molar volume of the solvent. According to Eq. 18 and the relative variation in molar volume-pressure curve, three interesting phenomena can be observed at any temperature: i) with increasing CO$_2$ mole fraction ($x_1$), liquid molar volume ($v$) decreases when ($\bar{v}_1 < \bar{v}_2$), ii) upon an increase in the high value of CO$_2$ mole fraction ($x_1$), liquid molar volume ($v$) reaches a minimum value at $\bar{v}_1 = \bar{v}_2$, and iii) by adding a very high value of CO$_2$ mole fraction ($x_1$), liquid molar volume ($v$) increases for $\bar{v}_1 > \bar{v}_2$ [50]. Fig. 1 shows the variation of the relative molar volume-pressure curve of the liquid phase in the binary system (CO$_2$-DMSO) at 308 K. It is evident that with increasing pressure, a minimum occurred in this curve at 7.27 MPa and a sharp increase of relative variation in molar volume can be observed above this point. Therefore, the optimum pressure for precipitating the solute in the GAS process will be larger than $P_{min}$ (7.27 MPa). The binary interaction parameters $k_{ij}$ and $l_{ij}$, shown in Eqs. 12 and 13, have been optimized using GA for each temperature. Table 3 shows these binary interaction parameters.

Fig. 2 shows the relative changes in the molar volume by variation of the CO$_2$ mole fraction at 308 K. As can be seen, with increasing the CO$_2$ mole fraction, a sharp drop can be detected in relative molar volume change until reaching the lowest value at carbon dioxide mole fraction of 0.714. At this point, by enhancing the CO$_2$ mole fraction of the liquid phase, the variation of relative molar volume showed a vertical increase. According to Figs. 1 and 2, at optimum operational pressure ($P_{min} = 7.27$ MPa), the CO$_2$ mole fraction is 0.714 in such a way that the change of relative molar volume shows a minimum value.

### Table 3. Binary interaction parameters from PR and the vdW2 mixing rules for the system CO$_2$ (1) + DMSO (2)

<table>
<thead>
<tr>
<th>$T$ (K)</th>
<th>$k_{12}$</th>
<th>$l_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>308</td>
<td>0.035</td>
<td>0.031</td>
</tr>
<tr>
<td>318</td>
<td>0.026</td>
<td>0.016</td>
</tr>
<tr>
<td>328</td>
<td>0.019</td>
<td>0.011</td>
</tr>
<tr>
<td>338</td>
<td>0.011</td>
<td>0.007</td>
</tr>
</tbody>
</table>
Fig. 1. Relative expansion of the liquid phase as a function of pressure, calculated according to PR for the systems DMSO–CO$_2$ at 308 K

Fig. 2. Relative expansion of the liquid phase as a function of CO$_2$ mole fraction in the liquid phase, calculated according to PR for the systems DMSO–CO$_2$ at 308 K

Fig. 3. Relative expansion of the liquid phase as a function of pressure, calculated according to PR for the systems DMSO–CO$_2$ at different temperature (308, 318, 328, and 338 K)

Fig. 3 presents the computational results for the binary system (DMSO+CO$_2$) at any temperatures. The minimum pressure showed an increase by temperature enhancement and $P_{\text{min}}$ was determined 7.27, 7.61, 7.95, and 8.27 MPa at 308, 318, 328, and 338 K, respectively.
Esfandiari et al. [31] calculated the relative variation in molar volume in terms of pressure based on the PR-LCVM equation for the DMSO-CO$_2$ system. The minimum pressure (Pmin) was 7, 7.74, and 8.5 MPa at 308, 313, and 319 K, respectively.

The vapor-liquid equilibrium of DMSO and CO$_2$ as a function of temperature and pressure is presented in Fig. 4. As can be observed, an increase in the pressure and temperature resulted in an increment in the CO$_2$ mole fraction of the liquid phase. The degree of GAS processability to precipitate nanoparticles depended on the extent of mixing of CO$_2$ with solvent. The miscibility of DMSO and CO$_2$ increased with pressure and temperature.

**Ternary System**

The ternary system comprising carbon dioxide-dimethyl sulfoxide–finasteride was studied in this work. The results of finasteride solubility at the liquid phase for this system are shown in Fig. 5. A slight increase can be observed in the finasteride solubility in the liquid phase within a narrow pressure range (~2.4-3.1 MPa) at 308 K. Based on Fig. 5, the minimum pressure in the curve of the finasteride mole fraction in the liquid phase occurred at 7.49 MPa. Also, the binary interaction parameters $k_{ij}$ and $l_{ij}$, for the ternary system shown in Eqs. 11 and 12, have been optimized using GA for each temperature. Table 4 shows these binary interaction parameters.

![Fig. 4](image-url)  
**Fig. 4.** Comparison of VLE of DMSO–CO$_2$ system calculated according to PR at 308, 318, 328, and 338 K
Fig. 5. The calculated solubility of FNS in the liquid phase expanded by supercritical CO$_2$ for the ternary system of CO$_2$-DMSO-finasteride at 308 K.

Table 4. Binary interaction parameters from PR and the vdW2 mixing rules for the system CO$_2$ (1) + DMSO (2) + FNS (3)

<table>
<thead>
<tr>
<th>T (K)</th>
<th>$k_{12}$</th>
<th>$k_{13}$</th>
<th>$k_{23}$</th>
<th>$l_{12}$</th>
<th>$l_{13}$</th>
<th>$l_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>308</td>
<td>0.1950</td>
<td>0.2300</td>
<td>0.0340</td>
<td>-0.0230</td>
<td>0.0500</td>
<td>0.2500</td>
</tr>
<tr>
<td>318</td>
<td>0.1190</td>
<td>0.2200</td>
<td>0.0328</td>
<td>-0.0240</td>
<td>0.0480</td>
<td>0.2300</td>
</tr>
<tr>
<td>328</td>
<td>0.0930</td>
<td>0.2100</td>
<td>0.0310</td>
<td>-0.0250</td>
<td>0.0460</td>
<td>0.2250</td>
</tr>
<tr>
<td>338</td>
<td>0.0900</td>
<td>0.1900</td>
<td>0.0300</td>
<td>-0.0260</td>
<td>0.0145</td>
<td>0.2200</td>
</tr>
</tbody>
</table>

Fig. 6 presents the relative molar volume variation of this system at 308 K. As shown in this figure, in the same pressure range a minimum can be detected, showing a sharp decrease in the finasteride concentration. This means that at this point ($P_{\text{min}} = 7.49$ MPa) almost all the solute is precipitated.

Fig. 6. Relative expansion of the liquid phase as a function of pressure, calculated according to PR for the systems DMSO–CO$_2$–FNS at 308 K.

Fig. 7 depicts the relative molar volume change-pressure curve for the ternary system at different temperatures. As can be seen, the minimum value of pressure increased by temperature elevation. The estimated $P_{\text{min}}$ were 7.49, 8.13, 8.51, and 9.03 MPa at 308, 318, 328, and 338 K, respectively. The modeling results for the minimum value of pressure in the binary and ternary systems are illustrated in Figs. 3 and 7. Accordingly, the value of $P_{\text{min}}$ in the binary system was smaller than that of the ternary system at a constant temperature. This indicates that the optimum operating pressure for the GAS process should be determined in such a way that to show the relative expansion of the liquid phase at least.
Fig. 7. Relative expansion of the liquid phase as a function of pressure, calculated according to PR for the systems $\text{CO}_2$–DMSO–FNS at different temperature (308, 318, 328, and 338 K)

**Conclusion**

In this study, the Peng–Robinson equation of state with vdW2 mixing rules was employed to determine the phase equilibrium data for the binary ($\text{CO}_2$+DMSO) and ternary ($\text{CO}_2$ +DMSO+FNS) systems at temperatures ranging from 308 to 338 K. The method of Marrero and Gani was proposed for calculating the critical properties of the drug. A new explanation of the relative variation in molar volume investigated by De la Fuente Badilla et al. [39] was also adapted for finding the optimum operating condition (T, P) for the GAS process. At certain temperatures, the optimum operating pressure for precipitation of finasteride particles was determined by plotting relative molar volume variation vs. pressure diagrams. This indicates that approximately 100% of FNS particles were precipitated at optimum operating conditions (P, T). The Pmin of the ternary system (DMSO-$\text{CO}_2$-FNS) was calculated 7.49, 8.13, 8.51, and 9.03 MPa at 308, 318, 328, and 338 K. By comparing the position of the pressure on the solubility curve of FNS in the liquid phase (Fig. 5) and the relative expansion of the liquid phase (Fig. 6), it can be found that the minimum pressure shown in these figures are the same (Pmin = 7.49 at T = 308 K). Moreover, the results showed that Pmin value increased by temperature enhancement.

**Nomenclature**

- $a(T)$: Energy parameter of the cubic EoS (J, m$^3$, mol$^{-2}$)
- $b$: Volume parameter for equations of state (m$^3$, mol$^{-1}$)
- $H_{\text{fusion}}$: Heat of fusion for the solute (J, mol$^{-1}$)
- $k_{ij}$: Binary interaction parameters in the mixing rules
- $l_{ij}$: Binary interaction parameters in the mixing rules
- $P$: Pressure (Pa)
- $P_c$: Critical pressure (Pa)
- $P_{\text{ref}}$: Reference pressure (Pa)
- $R$: universal Gas constant, $R=8.314$ (J, mol$^{-1}$, K$^{-1}$)
- $T$: Temperature (K)
- $T_b$: Boiling point temperature (K)
- $T_c$: Critical temperature (K)
- $T_r$: Reduced temperature
- $v$: Molar volume of solution (m$^3$, mol$^{-1}$)
- $\bar{v}_1$: Partial molar volume of CO$_2$ (m$^3$, mol$^{-1}$)
- $\bar{v}_2$: Partial molar volume of the solvent (m$^3$, mol$^{-1}$)
- vdW2: Van der Waals mixing rule with two adjustable parameters
x_i \quad \text{Liquid-phase mole fraction of the component } i
\y_i \quad \text{Vapor-phase mole fraction of the component } i
Z \quad \text{Compressibility factor}

\textit{Greek symbols}
\alpha(T) \quad \text{Temperature-dependent equation of state parameter}
\Delta \quad \text{Property change}
\phi \quad \text{Fugacity coefficient}
k \quad \text{Peng-Robinson equation parameter}
\omega \quad \text{Acentric factor}

\textit{Subscripts}
0 \quad \text{Reference condition}
1 \quad \text{Anti-solvent}
2 \quad \text{Solvent}
3 \quad \text{Solute}
c \quad \text{Critical property}
i, j \quad \text{Component}
L \quad \text{Liquid phase}
t_p \quad \text{Triple point}

\textit{Superscripts}
L \quad \text{Liquid phase}
S \quad \text{Solid-phase}
V \quad \text{Vapor phase}

\textbf{References}


[5] Esfandiar N. Production of micro and nano particles of pharmaceutical by supercritical carbon dioxide. The Journal of Supercritical Fluids. 2015 May 1;100:129-41.


[31] Esfandiari N, Ghoreishi SM. Optimal thermodynamic conditions for ternary system (CO₂, DMSO, ampicillin) in supercritical CO₂ antisolvent process. Journal of the Taiwan Institute of Chemical Engineers. 2015 May 1;50:31-6.


[46] Reid RC, Prausnitz JM, Poling BE. The properties of gases and liquids.


