Investigation of Pressure Pulse Distribution in Porous Media

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Abstract

Diffusivity equation commonly used for pressure distribution prediction in porous media results from substituting equation of state and continuity equation in Navier-Stokes momentum equation. From mathematical point of view this equation format shows infinite propagation speed for pressure pulse through porous media, which is physically impossible. This issue may caused by numerous assumptions that has been implemented for developing diffusivity equation. However, if we omit two main assumptions of steady state condition and constant velocity and consider linear approximation for velocity field, the pressure propagation differential equation would be hyperbolic which is called Telegraph Equation. The propagation speed is limited for this equation.

In this work, these equations are compared in prediction of pressure pulse propagation in Cartesian coordination with different parameters. The results show that the telegraph equation has minor correction in some cases as: far distances from pressure pulse source, when the fluid has high viscosity and for the rocks with low porosity and permeability; so considering common parameters in hydrocarbon reservoirs, the diffusivity equation has sufficient accuracy for reservoir engineering applications.

Keywords: Diffusivity equation, Darcy equation, Pressure distribution equation, Porous media, Telegraph equation

Introduction

The diffusivity equation is applying for prediction of pressure pulse distribution in porous media by reservoir engineers. This equation is developed by using numerous simplifying assumption and substitution of equation of state and continuity equation in Navier-Stokes momentum equation. The mathematical form of this equation shows infinite propagation speed for pressure pulse through porous media, which is physically impossible. In the other words, based on the equation once the pressure pulse influences one point of the porous media, it would be entirely impressed. Since the fluid velocity is not higher than the velocity of sound, this phenomenon is physically meaningless and could not be expected a pressure pulse to influence a point earlier than this time. This problem may arise from various simplified assumptions used in the development of the equation. Several attempts are done to eliminate some of these assumptions. In this work, attempts that led to equations with limited propagation velocity are reviewed and then their behavior is compared with diffusivity equation.

Review of pressure distribution equations and their development

Diffusivity equation that is also known as Darcy equation was used for describing the steady state and laminar motion of Newtonian fluid that flow in a rigid medium in isothermal conditions. In this equation, fluid is assumed to be single phase and no slippage occurs between rock and the fluid. The fluid and solid phases are chemically neutral so there is no attraction, departure or molecular forces in between. This equation is shown as 1.

$$\nabla.(\mathrm{k}\,\nabla \mathrm{P}) = \phi C_t \mu \frac{\partial P}{\partial \mathrm{t}} \qquad (1)$$

The diffusivity equation is a parabolic second order differential equation with infinite propagation speed of pressure pulse through porous media, which is physically impossible. This error may be caused by numerous assumptions that have been implemented for developing diffusivity equation.

This form of second order differential equation is used for description of various

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physical properties such as mass transfer and electromagnetic although the intrinsic problem of infinite speed, always make it physically meaningless. In divergent sciences, different attempts are done to solve this restriction. For example, Joseph B. Keller [1] explains that the rate of component diffusion could not be more than one single molecule velocity so the infinite predicted velocity from diffusivity equation is not correct. He solved this paradox by using random walk concept and presents a new equation with finite propagation velocity for description of mass transfer. The equation that includes second order time derivative in addition to first order derivative is known as Telegraph equation. Mathematically, this is a hyperbolic second order partial differential equation with finite propagation speed.

After presenting Darcy equation and considering various simplified assumptions used in development of the equation, several attempts are done to eliminate some of assumptions and adapting theoretical equation with experimental results. For the first time Forcheirmer amend this equation by adding higher order term to the primary equation in 1782 [2]. These statements are expected to be appeared since the underlying microscopic equations of momentum balance are themselves nonlinear in the point velocity field. The term $\rho\vec{\upsilon}\vec{\upsilon}$, which represents the convective flux of momentum density, appears in the momentum balance equation. Just in parallel streamlines, the divergence of this tensor vanishes. The steady flow streamlines in most porous media are not parallel; so the nonlinear dependence of the pressure gradient appears. This nonlinearity is not dependent on the change of velocity field but just because of diverging and converging streamlines. Klinkenberg demonstrated that the permeability coefficient in Darcy's law depends on the absolute pressure or, alternatively, on the density field [3]. However, because he neglected inertial terms of the Forcheirmer type, his correction coefficient could not be represented by a constant but tended toward a constant as the velocity decreased. Forcheirmer and Klinkenberg modification can be amalgamate to model both inertia and slip during steady flow. Fatt suggested that the cause of deviations of pressure transient data from the prediction of Darcy equation could be not only from selection of Darcy equation but also because of existence of dead-end pores [4]. On the other hand, Oroveanu and Pascal noted that the time derivative of the momentum density must be included in the equation of motion since it measures the local rate of momentum density variations. Their differential equation for pressure is the telegraph equation. However, the form of this equation predicts that the speed of pressure propagation through the pore structure is the same as that of the bulk fluid [5]. M. K. Hubbert attempted a derivation of Darcy's law by volume averaging the Navier-Stokes equations. Since these equations represent momentum balance at a point within an open set of points containing the fluid itself, Hubbert's volume averaging cannot lead to terms involving transfer of momentum between the fluid and the walls of the pores. Once these viscous-tractions are lost by choosing a control volume containing only the fluid, they cannot be recovered by averaging the limiting point equations.

In 1967, Foster and Mc Millen developed complete averaging of linear momentum balance equation for a single-phase fluid that flows through homogeneous, incompressible and porous media [7]. In this study, two main assumption of Darcy equation for Newtonian fluids are omitted. Main parts of the development path of this equation are presented as follow:

The equations of linear momentum balance at a point in a continuum are:

$$\frac{\partial\rho\vec{v}}{\partial t} + \nabla.\rho\vec{v}\vec{v} = -\rho g\vec{k} + \nabla.\vec{\vec{\tau}} \qquad (2)$$

Where ρ is density field, $\vec{v}$ is fluid velocity field, ∇ is gradient operator, $\vec{\vec{\tau}}$ is stress tensor, $\vec{k}$ is unit vector directed upward and g is gravitational acceleration. For our purpose, consider a homogeneous fluid in a porous medium and imagine a region closed by the stationary surface S. let $\vec{r}$ denote some representative point in this region and let V denote the volume of fluid contained within the region. Eq. 2 is first averaged over volume V,

$$\frac{1}{V}\iiint_V \frac{\partial\rho\vec{v}}{\partial t}\, dv + \frac{1}{V}\iiint_V \nabla.\rho\vec{v}\vec{v}\, dv = -\frac{g\vec{k}}{V}\iiint_V \rho\, dv + \frac{1}{V}\iiint_V \nabla.\vec{\vec{\tau}}\, dv \quad (3)$$

Since the integral is independent of time the first term from right and the left side of the equation is simplified to time derivative of the Integrate value limited within the S. No loss in generality results if this momentum density and body force density is now associated at the representative point $\vec{r}$ at time t. By the theorem of Gauss, the volume integrals involving divergences can be transformed into integrals over the surface, which bound the fluid. These surfaces are of two kinds: first, surface passes through pored containing fluid S_e and second the pore walls of the rock volume within the S, S_i . Where < > implies a volume average. Eq. 3 may thus be expressed as

$$\frac{\partial\langle\rho\vec{v}\rangle}{\partial t} + \frac{1}{V}\iint_{S_e} d\vec{S_e}.\rho\vec{v}\vec{v} + \frac{1}{V}\iint_{S_i} d\vec{S_\imath}.\rho\vec{v}\vec{v} = -\langle\rho\rangle g\vec{k} + \frac{1}{V}\iint_{S_e} d\vec{S_e}.\vec{\vec{\tau}} + \frac{1}{V}\iint_{S_i} d\vec{S_\imath}.\vec{\vec{\tau}} \quad (4)$$

Providing there is no slip at the pore walls, the integral of $\rho\vec{v}\vec{v}$ over the external surface may be expressed again as volume integrals, i.e.

$$\frac{1}{V}\iint_{S_e} d\vec{S_e}.\rho\vec{v}\vec{v} = \frac{1}{V}\nabla.\iiint_V \rho\vec{v}\vec{v}\, dv = \nabla.\langle\rho\vec{v}\vec{v}\rangle \quad (5)$$

$$\frac{1}{V}\iint_{S_e} d\vec{S_e}.\vec{\vec{\tau}} = \frac{1}{V}\nabla.\iiint_V \vec{\vec{\tau}}\, dv = \nabla.\langle\vec{\vec{\tau}}\rangle \quad (6)$$

Therefore, we can rewrite the equation as follow:

$$\frac{\partial\langle\rho\vec{v}\rangle}{\partial t} + \nabla.\langle\rho\vec{v}\vec{v}\rangle = -\langle\rho\rangle g\vec{k} + \nabla.\langle\vec{\vec{\tau}}\rangle + \frac{1}{V}\iint_{S_i} d\vec{S_\imath}.\vec{\vec{\tau}} \quad (7)$$

We choose to study the compressible Newtonian fluid. The stress tensor for this fluid has the following form.

$$\vec{\vec{\tau}} = \mu(\nabla\vec{v} + (\nabla\vec{v})^T) + (\eta - {}^2/_3\,\mu)(\nabla.\vec{v})\,\vec{\vec{\imath}} - \nabla p\left(\mathrm{Tr}\,\vec{\vec{v}}\right)\vec{\vec{\imath}} \quad (8)$$

That in the equation μ is shear viscosity coefficient, η is bulk viscosity coefficient, $\vec{\vec{\imath}}$ is two dimensional unit tensor, $(\nabla\vec{v})^T$ is transpose of $\nabla\vec{v}$, $\vec{\vec{\square}}$ is strain tensor, Tr $\vec{\vec{\square}}$ is trace of strain tensor (sum of diagonal elements) and P is pressure. Considering more details that is presented in [7] the average stress tensor is as follow:

$$\nabla.\langle\vec{\vec{\tau}}\rangle = \mu\nabla^2\langle\vec{v}\rangle + (\eta + {}^1/_3\,\mu)\nabla(\nabla.\langle\vec{v}\rangle) - \nabla\langle P\rangle \quad (9)$$

Next, the integral must be evaluated over the internal surface. Let $\vec{n}$ be the unit normal at a point on the internal surface, reckoned positive pointing into the fluid, and let n measure distance along this vector. It follows that $d\vec{S_\imath} = -dS_i\vec{n}$. We have at any point on the internal surface:

$$\nabla.\vec{v}|_{S_i} = \frac{\partial v_n}{\partial n}\Big|_{S_i} \quad (10\text{-}1)$$

$$\nabla\vec{v}|_{S_i} = \vec{n}\frac{\partial\vec{v}}{\partial n}\Big|_{S_i} \quad (10\text{-}2)$$

$$(\nabla\vec{v})^T|_{S_i} = \frac{\partial\vec{v}}{\partial n}\vec{n}\Big|_{S_i} \quad (10\text{-}3)$$

Denoting $\frac{S_i}{V}$ by 1/L (L being a characteristic length of the medium) the equation of 8 can be rewrite as follow:

$$\frac{1}{LS_i}\int_{S_i} d\vec{S_\imath}.\vec{\vec{\tau}} = -\frac{1}{L}\left\{\frac{\mu}{S_i}\int_{S_i} d\vec{S_\imath}\frac{\partial\vec{v}}{\partial n} + \frac{\mu}{S_i}\int_{S_i} d\vec{S_\imath}\frac{\partial v_n}{\partial n}\vec{n} + \frac{(\eta - {}^2/_3\,\mu)}{S_i}\int_{S_i} d\vec{S_\imath}\frac{\partial v_n}{\partial n}\vec{n} - \frac{1}{S_i}\int_{S_i} d\vec{S_\imath}\, p\,\vec{n}\right\} = -\frac{1}{L}\left\{\mu\left\langle\frac{\partial\vec{v}}{\partial n}\right\rangle_{S_i} + (\eta + {}^1/_3\,\mu)\left\langle\frac{\partial v_n}{\partial n}\right\rangle_{S_i} - \langle p\,\vec{n}\rangle_{S_i}\right\} \quad (11)$$

The average equations of hydrostatic balance are obtained only if the normal surface force $\langle p\,\vec{n}\rangle$ averaged over the internal surface vanishes. This term should also vanish during fluid motion since otherwise traction independent of viscosity appears. Combining equations 9 and 11 with equation 7,

$$\frac{\partial\langle\rho\vec{v}\rangle}{\partial t} + \nabla.\langle\rho\vec{v}\vec{v}\rangle = -\langle\rho\rangle g\vec{k} + \mu\nabla^2\langle\vec{v}\rangle + \left(\eta + {}^1\!/_3\,\mu\right)\nabla(\nabla.\langle\vec{v}\rangle) - \nabla\langle P\rangle - \frac{\mu}{L^2}\langle\frac{\partial\vec{v}}{\partial L_D}\rangle_{S_i} - \left(\frac{\eta+{}^1\!/_3\,\mu}{L^2}\right)\langle\frac{\partial v_n}{\partial L_D}\rangle_{S_i} \quad (12)$$

Where L_D is the dimensionless distance n/L. With the exception of the last two terms on the right-hand side, equation 12 is formally identical to the Novier-Stokes equations and, if the control volume shrinks to a point within a pore, two equations converge.

Steady motion

For steady flows in homogenous porous media, the derivative of the product of momentum and the density field vanishes and any change in velocity would be neglected. The only remaining term in equation 12 is as equation 13.

$$0 = -\langle\rho\rangle g\vec{k} - \nabla\langle P\rangle \quad (13)$$

But if we just neglect second order derivative and v_n gradient, the equation would be as 14.

$$0 = -\langle\rho\rangle g\vec{k} - \nabla\langle P\rangle - \frac{\mu}{L^2}\langle\frac{\partial\vec{v}}{\partial L_D}\rangle_{S_i} \quad (14)$$

The quantity $\langle\frac{\partial\vec{v}}{\partial L_D}\rangle_{S_i}$ is a vector with the dimension of velocity. Since one vector can always be mapped into another by a 2-tensor, we may write

$$\langle\frac{\partial\vec{v}}{\partial L_D}\rangle_{S_i} = \varphi\vec{\vec{T}}.\langle\vec{v}\rangle \quad (15)$$

Here φ is the void fraction and $\varphi\,\langle\vec{v}\rangle$ is the usual Darcy velocity. Thus, the momentum balance for a steady flow is

$$-\nabla\langle P\rangle = -\langle\rho\rangle g\vec{k} - \frac{\varphi\mu\vec{\vec{T}}}{L^2}\langle\vec{v}\rangle_{S_i} \quad (16)$$

where $\frac{\vec{\vec{T}}}{L^2} = \vec{\vec{R}}$ is the resistivity tensor of the medium. So, the equation 16 will be converted to 17. The permeability K is the reciprocal of R.

$$-\nabla\langle P\rangle = \langle\rho\rangle g\vec{k} + \varphi\mu\vec{\vec{R}}.\langle\vec{v}\rangle = \langle\rho\rangle g\vec{k} + \frac{\varphi\mu\,.\langle\vec{v}\rangle}{\vec{\vec{K}}} \quad (17)$$

In homogenous media $\vec{\vec{R}}$ is equal to its multiple of the unit 2-tensor and the traction always parallels the average velocity. So $\vec{\vec{R}} = R\vec{\vec{I}}$ where R is a constant value; so:

$$-\nabla\langle P\rangle = \langle\rho\rangle g\vec{k} + \varphi\mu\langle\vec{v}\rangle = \langle\rho\rangle g\vec{k} + \frac{\varphi\mu}{k}\langle\vec{v}\rangle \quad (18)$$

Equation 17 becomes the generalized form of Forcheirmer's law when R or the components of $\vec{\vec{R}}$ are expanded in a Taylor's series in the powers of $|\langle\vec{v}\rangle|$. Furthermore, the possible dependence of R and the components of $\vec{\vec{R}}$ on $\langle\rho\rangle$ provide a basis for the Klinkenberg correction.

Non-steady motions

In contrast to the relatively simple form that equation 18 takes for steady flows, in principle all terms must be considered for the more general situation. It becomes important to exercise caution in neglecting parts of the momentum balance. However, for petroleum reservoir porous media the small value of L insures that the viscous traction will continue to dominate equation 12. For instance the term of $\mu\nabla^2\langle\vec{v}\rangle$ is of the order of $\frac{\mu\vec{\vec{T}}}{L^2}\langle\vec{v}\rangle$ just if the velocity varies in the order of $\langle\vec{v}\rangle$ over a distance comparable to L. For most transient flows no such abrupt changes occur. Terms in the spatial derivatives of velocity and those arising from dilatation are again neglected since these are small if the fluid is only slightly compressible or is a perfect gas.

$$\frac{\partial\langle\rho\vec{v}\rangle}{\partial t} = -\langle\rho\rangle g\vec{k} + -\nabla\langle P\rangle - \frac{\mu}{L^2}\langle\frac{\partial\vec{v}}{\partial L_D}\rangle_{S_i} \quad (19)$$

Now, two vector fields, $\frac{\partial\langle\rho\vec{v}\rangle}{\partial t}$ and $\langle\vec{v}\rangle$ exists in the equation as well as the scalar field $\langle\rho\rangle$. Writing the $\langle\frac{\partial\vec{v}}{\partial L_D}\rangle_{S_i}$ in the direction of, $\frac{\partial\langle\rho\vec{v}\rangle}{\partial t}$ and $\langle\vec{v}\rangle$ by 2-tenssor of $\vec{\vec{T}}$ and $\vec{\vec{W}}$ those could be the function of $\langle\rho\rangle$, $\langle\vec{v}\rangle$, $\frac{\partial\langle\rho\vec{v}\rangle}{\partial t}$ and $\frac{\partial\langle\rho\vec{v}\rangle}{\partial t}$:

$$\langle\frac{\partial\vec{v}}{\partial L_D}\rangle_{S_i} = \varphi\vec{\vec{T}}.\langle\vec{v}\rangle + \vec{\vec{W}}.\frac{\partial\langle\rho\vec{v}\rangle}{\partial t} \quad (20)$$

By substituting 20 in 19 and rearranging we have:

$$\left(\vec{\vec{T}} + \frac{\mu}{L^2}\vec{\vec{W}}\right)\frac{\partial\langle\rho\vec{v}\rangle}{\partial t} = -\langle\rho\rangle g\vec{k} + -\nabla\langle P\rangle - \frac{\varphi\mu\vec{\vec{T}}}{L^2}\langle\vec{v}\rangle \quad (21)$$

For an isotropic medium, just main values of T and W are considered and 12 results. Just in case of small values of $\langle\vec{v}\rangle$ and $\frac{\partial\langle\rho\vec{v}\rangle}{\partial t}$ and when T and W are the week function of ρ we can make this assumption.

$$\left(T + \frac{\mu}{L^2}W\right)\frac{\partial\langle\rho\vec{v}\rangle}{\partial t} = -\langle\rho\rangle g\vec{k} + -\nabla\langle P\rangle - \frac{\varphi\mu T}{L^2}\langle\vec{v}\rangle \quad (22)$$

Letting v_f denote the speed of sound through the fluid in question and $\langle\rho\rangle_0$ the average density of some standard state, we then set N_A the dimensionless number, which reflects the topology of the pore structure.

$$\frac{\mu W}{L^2} = \frac{\mu}{\langle\rho\rangle_0 k^{\frac{1}{2}} V_f} N_A \quad (23)$$

T/L^2 is also replaced by R=1/k and for the sake of brevity, let $\Psi = 1 + \left(\frac{\mu}{\langle\rho\rangle_0 k^{\frac{1}{2}} V_f} N_A\right)$ the equation of 22 is converted to 24.

$$\frac{\partial\langle\rho\vec{v}\rangle}{\partial t} = -\nabla\langle P\rangle - \frac{\varphi\mu T}{L^2}\langle\vec{v}\rangle \quad (24)$$

In this stage using continuity equation and equation of state $\langle\rho\vec{v}\rangle$ is omitted and the equation of 25 results.

$$\psi\frac{\partial^2\langle P\rangle}{\partial t^2} = \nabla^2\langle P\rangle + \frac{\varphi\mu}{k}\nabla.\langle\vec{v}\rangle \quad (25)$$

To proceed further a series of approximations must be made. Assume that $\langle P\rangle$ and $\langle\vec{v}\rangle$ are almost statistically uncorrelated and using mathematical operation that is presented in [7] the following equations will be result for slightly compressible and perfect gas.

$$\psi\, C\,\langle\rho\rangle_0\frac{\partial^2\langle P\rangle}{\partial t^2} + \frac{\varphi\mu C}{k}\frac{\partial\langle P\rangle}{\partial t} = \nabla^2\langle P\rangle \quad (26)$$

$$\psi\, Y\frac{\partial^2\langle P\rangle}{\partial t^2} + \frac{\varphi\mu}{k\langle P\rangle_0}\frac{\partial\langle P\rangle}{\partial t} = \nabla^2\langle P\rangle \quad (27)$$

However, since $Y = \frac{\langle\rho\rangle_0}{\langle P\rangle_0}$ and C are compressible, the groups $\frac{k}{\varphi\mu C}$ and $\frac{k\langle P\rangle_0}{\varphi\mu}$ have the dimensions of diffusion coefficients, which are $\frac{L^2}{t}$. The groups $\psi\, C\,\langle P\rangle_0$ and $\psi\, Y$ have dimensions of reciprocal velocity squared, which are $\frac{t^2}{L^2}$.

Denoting the general diffusion coefficient by D and the general velocity by v_a, the equations will become:

$$\frac{1}{v_a{}^2}\frac{\partial^2\langle P\rangle}{\partial t^2} + \frac{1}{D}\frac{\partial\langle P\rangle}{\partial t} = \nabla^2\langle P\rangle \quad (28)$$

$$D = \begin{cases} \frac{k}{\varphi\mu C} & \text{for slightly compressible liquid} \\ \frac{k\langle P\rangle_0}{\varphi\mu} & \text{for perfect gases} \end{cases}$$

$$\frac{1}{v_a{}^2} = \begin{cases} \psi\, C\,\langle\rho\rangle_0 & \text{for slightly compressible liquid} \\ \psi\, Y & \text{for perfect gases} \end{cases}$$

The velocity of sound through a slightly compressible gas is as equation 29 and the isothermal velocity of sound through a perfect gas is as equation 30. Therefore, the physical interpretation of $\frac{v_f{}^2}{v_a{}^2} = \psi$ is the ratio of the speed of sound in a bulk fluid to the speed of sound through the pore structure of a porous medium containing this fluid.

$$V_L = \frac{1}{(C\,\rho_0)^{\frac{1}{2}}} \quad (29)$$

$$V_g = \frac{1}{Y^{\frac{1}{2}}} \quad (30)$$

The result equation is a special form of hyperbolic equation that is known as telegraph equation. Comparing the equation with the general form of hyperbolic equation, it is found that the velocity of pressure pulse penetration is limited and equal to v_a. Therefore, it can be seen that Foster removed the infinite propagation speed in diffusivity equation by omission of two main assumptions of Darcy equation.

Comparison of diffusivity equation and telegraph equation

In the next step after analytical solution of both equations in Cartesian coordination, their prediction of pressure distribution is compared.

The one-dimensional equation of diffusivity in Cartesian coordination and its boundary condition is as follow:

$$\frac{\partial^2 \Delta P}{\partial x^2} = \frac{1}{D}\frac{\partial \Delta P}{\partial t} \tag{31}$$

BC: $\Delta P = 0 \ , t = 0$

$$t > 0 \quad \begin{cases} \Delta P \to 0 & , x \to \infty \\ \Delta P = P_0 & , x = 0 \end{cases}$$

The solution of this equation is possible with different methods such as separation parameters and Laplace transforms and would be as equation 32.

$$\Delta P = P_0 \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) \tag{32}$$

Now, we consider one dimensional telegraph equation in Cartesian coordination and its boundary condition.

$$\frac{\partial^2 \Delta P}{\partial x^2} = \frac{1}{v_a^2}\frac{\partial^2 \Delta P}{\partial t^2} + \frac{1}{D}\frac{\partial \Delta P}{\partial t} \tag{33}$$

BC: $\Delta P = 0 \ , \frac{\partial \Delta P}{\partial t} = 0, t = 0$

$$t > 0 \quad \begin{cases} \Delta P \to 0 & , x \to \infty \\ \Delta P = P_0 & , x = 0 \end{cases}$$

The solution of this equation is possible via Laplace transform and Riemann equation and is as equation 34.

$$\Delta P(x,t) = P_0 H\left(t - \frac{x}{v_a}\right)\left\{ e^{-\frac{x v_a}{2D}} + \frac{x v_a}{2D}\int_{\frac{x}{v_a}}^{t} \frac{e^{-\frac{v_a^2 t}{2D}} Y_1\left(\frac{v_a^2}{2D}\sqrt{t^2 - \frac{x^2}{v_a^2}}\right)}{\sqrt{t^2 - \frac{x^2}{v_a^2}}} dt \right\} \tag{34}$$

Where Y_1 is Bessel function of the second kind of first order and $H(t - \frac{x}{v_a})$ is Heaviside function defined by

$$H\left(t - \frac{x}{v_a}\right) = \begin{cases} 0 \ , t < \frac{x}{v_a} \\ 1 \ , t > \frac{x}{v_a} \end{cases} \tag{35}$$

Thus, the solution as given by equation 34 shows that a time lag exists between initiating a disturbance and its arrival at a down-stream point x. This time lag is given by $\frac{x}{v_a}$. It is thus quit clear that the quantity v_a is indeed the propagation speed of pressure pulse through the pores of the medium.

For using telegraph equation in addition to the ordinary known parameters it is necessary to know the propagation speed of sound in porous media v_a. Foster in [8] addressed the result of experiments that is done in 6 different reservoir rock sample with gas as fluid sample. He calculated the v_a for each test. The speed of sound is change from 20 to 1600 cm/sec in different experiments. In this work we use these values for comparing the solution of two equations in different conditions. The experimental result from [8] is presented in table (1).

Table 1: Tests condition and results from [8]

Test	K (cm^2)	Porosity %	Viscosity(poise)	V_a(cm/sec)	t(sec)	Average pressure (dyne/cm^2)	Core lengh(cm)
1	0.47E-8	21	1.84E-4	210	0.005	1.083E6	7.6
2	2.29E-8	24.1	1.84E-4	538	0.005	1.083E6	7.7
3	10.22E-8	30.8	1.84E-4	1390	0.003	1.083E6	11.4
4	4.68E-8	21.6	1.84E-4	1560	0.001	1.083E6	6.9
5	1.06E-8	19.9	1.84E-4	492	0.003	1.083E6	5.5
6	1.05E-8	13.3	1.84E-4	206	0.007	1.083E6	5.5

In the first stage the solution of both, telegroph and Darcy, equation is presented in the lentgh of the core at a difined moment.

Figures (1) to (6) are presents the pressure profile in the lentgh of the cores which their parameters are presented in table (1). In all the tests the existance of Heaviside equation in telegroph equation cause the pressure propagation front to be specified like a jump in the profile. However the Darcy equation impress whole core length in the earliest moment.

The speed of propagation in the 2nd test is more than 1st test this might be because of higher permeability of the rock. This case is also applicable for the 3rd test with the highest value of rock permeability and high propagation speed. In the 4th test although the rock permeability is lower than the 3rd test but because of condensed rock with lower porosity the propagation speed is higher. In this test considering the high propagation speed the pressure profile is shown at t=0.001.

In the second stage the outlet pressure of a core, that is calculated via both equations, is shown passing time after applying a pressure accretion from 0 to 31 psi in the inlet of the core to demonstrate the effect of changing different parameters.

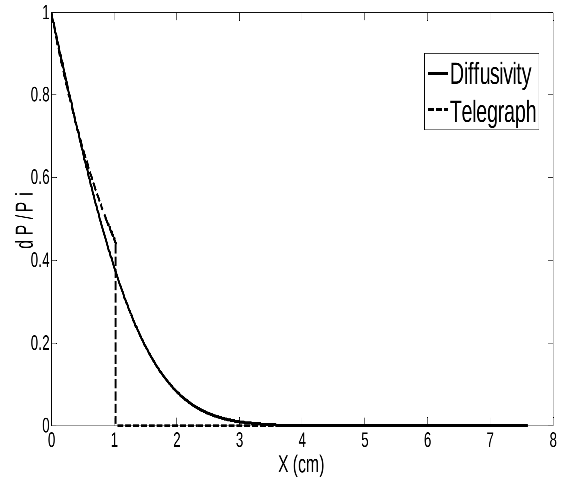

Figure 1:Comparsion of diffusivity and telegroph equation for test 1

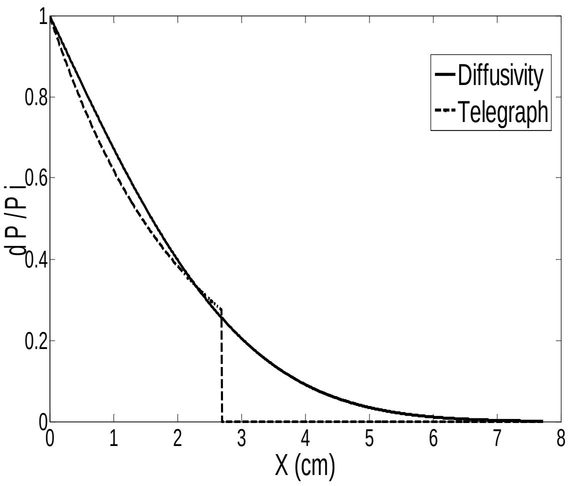

Figure 2: Comparsion of diffusivity and telegroph equation for test 2

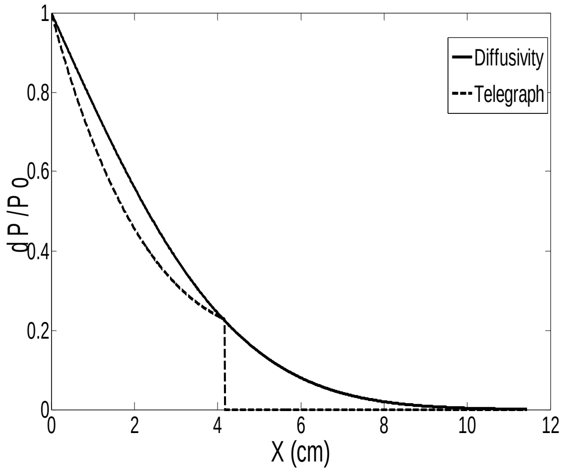

Figure 3: Comparsion of diffusivity and telegroph equation for test 3

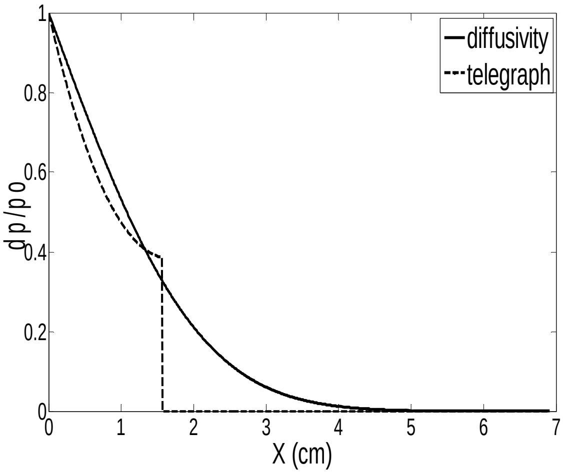

Figure 4: Comparsion of diffusivity and telegroph equation for test 4

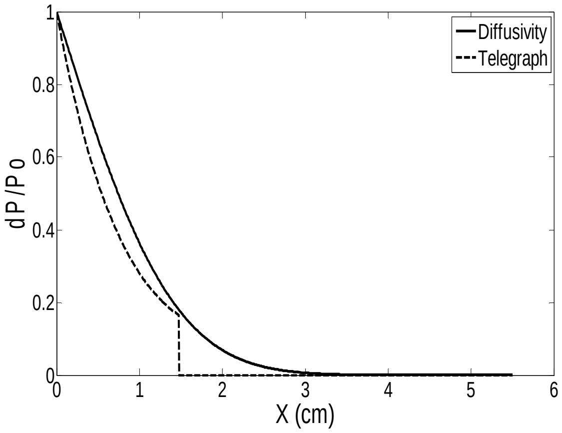

Figure 5: Comparsion of diffusivity and telegroph equation for test 5

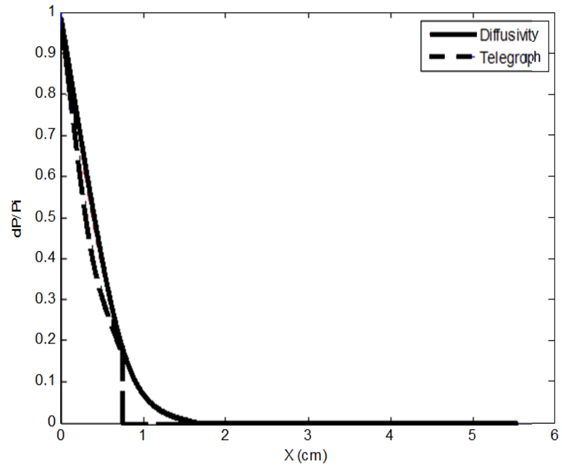

Figure 6: Comparsion of diffusivity and telegroph equation for test 6

Core length

Figures (7-1) and (7-2) are showing the outlet pressure of a core calculation via both Darcy and telegraph equations in two different cores with length of 11 and 50 cm, respectively. Other parameters those are the same for both cores are presented in table (2). Figure (7-3) is demonstrating the difference between two equations for each core. Rumination the results of equations indicates that two equations are approach together as the core length enlarged. So we can say that the error of using Darcy equation is going down as the distance rises. And also it express that the correction of telegraph equation become insignificant as time passes.

Table 2: Applied parameters for investigation of core length effect on pressure distribution

K	10 d
Porosity	30 %
Viscosity	0.02 cp
L	10 , 50 cm
Initial Pressure (psi)	0
Pressure Difference (psi)	31

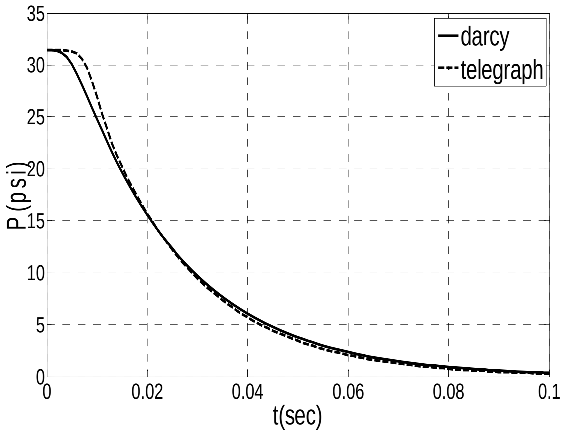

Figure 7-1: Comparsion of diffusivity and telegroph equation in 10 cm core

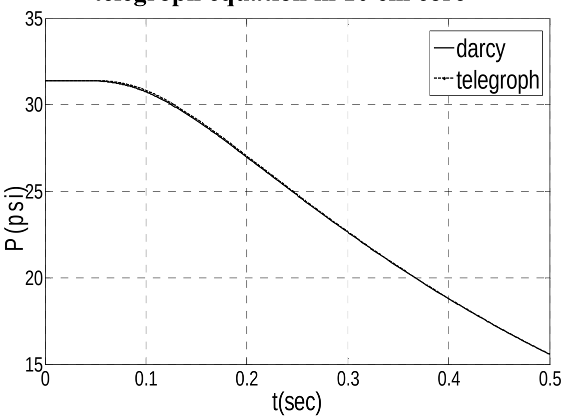

Figure 7-2: Comparsion of diffusivity and telegroph equation in 50 cm core

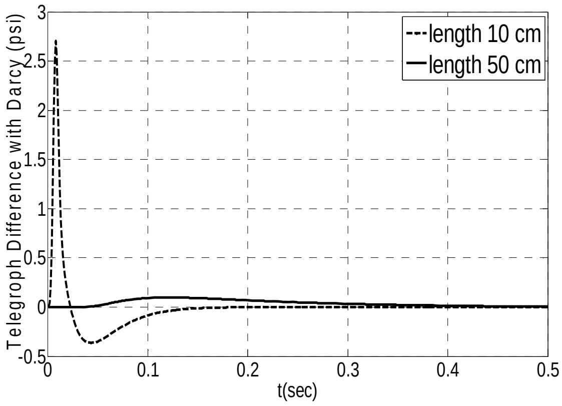

Figure 7-3: Diffusivity and telegroph equation differece in 10 cm core and 50 cm core

Permeability

Figures (8-1) and (8-2) are showing the outlet pressure of a core calculation via both Darcy and telegraph equations in two different core permeabilities 100 md and 1D, respectively. Other parameters those are the same for both cores are presented in table (3). Figure (8-3) is demonstrating the difference between two equations for each core. Rumination the results of equations indicates that the results of two equations are approach together as the permeability decrease. So we can say that the error of using Darcy equation is low in the petroleum reservoirs permeabilities. As we also saw in experimental results the propagation speed is higher in high permeable rocks. The effect of passing time can be seen in these curves too.

Table 3: Applied parameters for investigation of core permeability effect on pressure distribution

K	100 md , 1d
Porosity	30 %
Viscosity	0.02 cp
L	10 cm
Initial Pressure (psi)	0
Pressure Difference (psi)	31

Table 4: Applied parameters for investigation of core porosity effect on pressure distribution

K	1d
Porosity	10% ,30 %
Viscosity	0.02 cp
L	10 cm
Initial Pressure (psi)	0
Pressure Difference (psi)	31

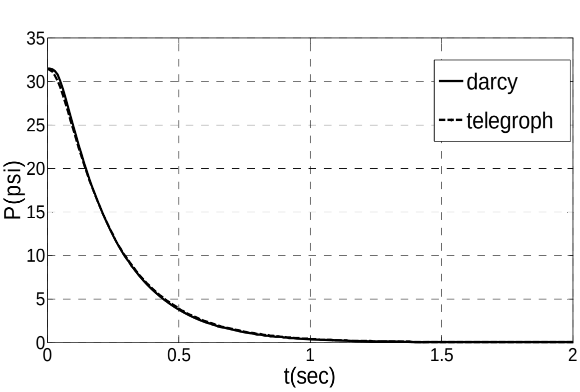

Figure 8-1: Compersion of diffusivity and telegroph equation for k=1D

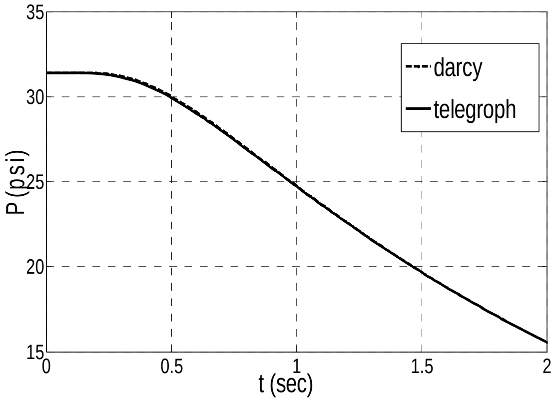

Figure 8-2: Compersion of diffusivity and telegroph equation for k=100md

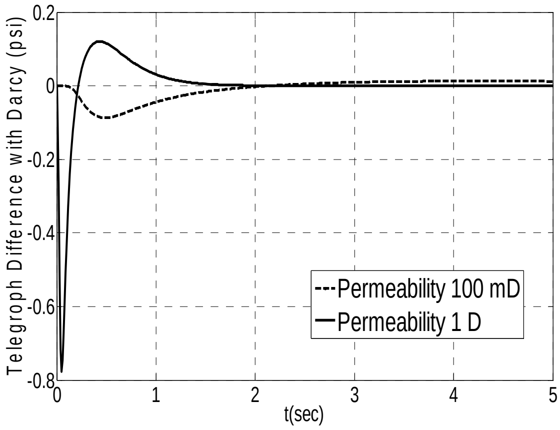

Figure 8-3: Diffusivity and telegroph equation differece for k=100md and k=1D

Porosity

Figures (9-1) and (9-2) are showing the outlet pressure of a core calculation via both Darcy and telegraph equations in two different core porosities 30% and 10%, respectively. Other parameters which are the same for both cores, are presented in table (4). Figure (8-3) is demonstrating the difference between two equations for each core. It is clear that the error of using Darcy equation decreases as porosity increases.

Viscosity

In the last part in figures (10-1) to (10-3), the effect of viscosity is investigated on the behavior of Darcy and telegraph equations that shows the difference between two equations is ample in low viscosity fluids. In the other word, the error of Darcy equation is more severe in gas fields.

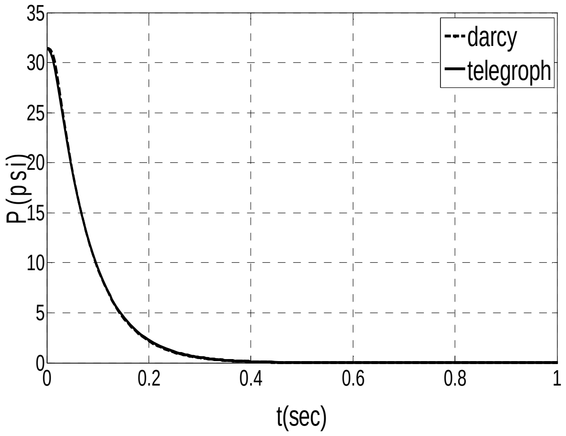

Figure 9-1: Compersion of diffusivity and telegroph equation for φ=30%

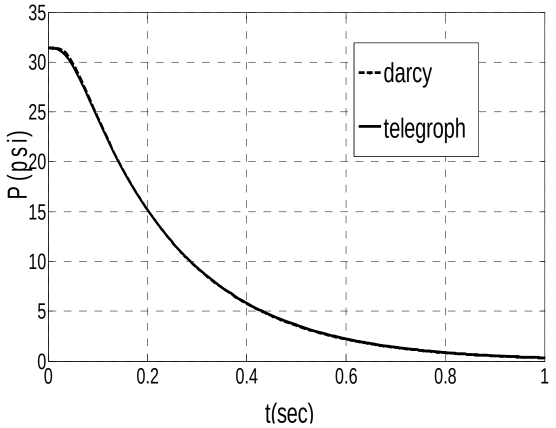

Figure 9-2: Compersion of diffusivity and telegroph equation for φ=10%

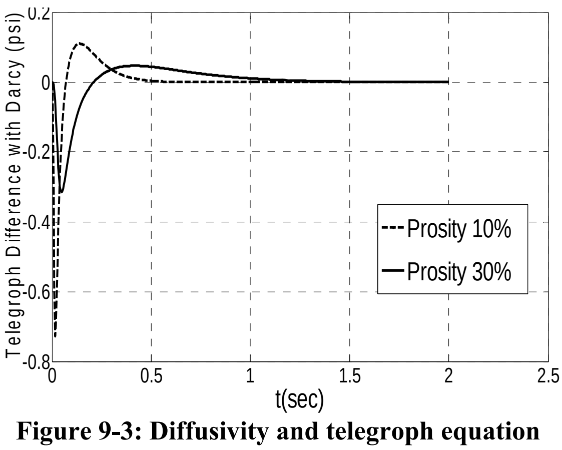

Figure 9-3: Diffusivity and telegroph equation differeces for φ=10% and φ=30%

Table 5: Applied parameters for investigation of fluid viscosity effect on pressure distribution

K	1d
Porosity	30 %
Viscosity	0.01 , 0.1 cp
L	10 cm
Initial Pressure (psi)	0
Pressure Difference (psi)	31

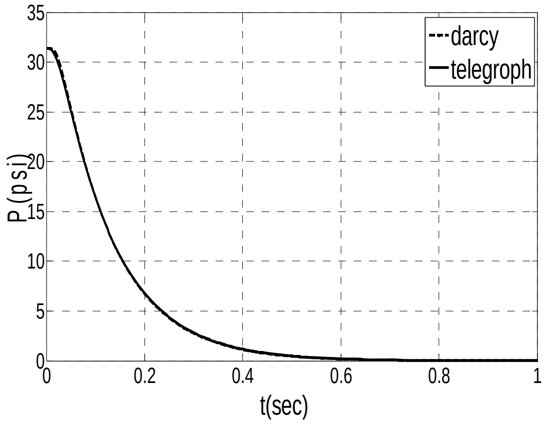

Figure 10-1: Compersion of diffusivity and telegroph equation for μ=0.01 cp

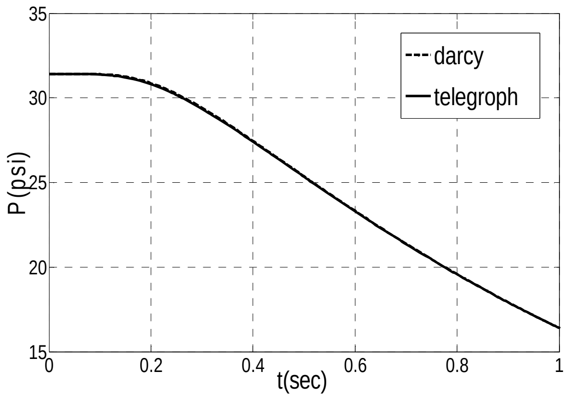

Figure 10-2: Compersion of diffusivity and telegroph equation for μ=0.1 cp

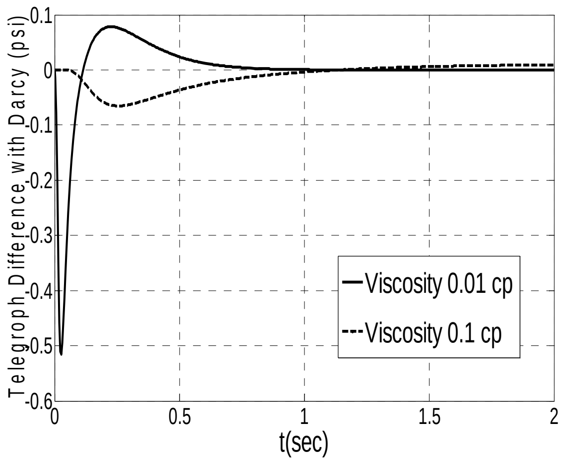

Figure 10-3: Diffusivity and telegroph equation differece for μ=0.1 cp and μ=0.01 cp

Conclusion

The Darcy diffusivity equation that is used to anticipate pressure distribution in porous media has the limitation of infinite propagation speed of pressure pulse from mathematical point of view. In fact, this is physically impossible since the movement of pressure pulse cannot be faster than fluids molecules speed. This limitation may caused by several simplifying assumtions that have applied during Darcy equation development. Neveretheless, with omittion of two main asumptions of steady state condition and constant velocity and considering linear approximation for velocity field the pressure propagation differential equation would be hyperbolic equation calls Telegraph equation. The propagation speed is limitted for this equation and is equal to sound velocity that is theoritically is equal to maximum fluid velocity.

In this work, the results of these equations which are compared with a step function are put into a simulation core. In the other word, the pressure is dropped from P_0 to zero. The investigations are based on analytical solution of the differential equation in Cartesian coordination. The experimental parameter of sound velocity through porous media is taken from already performed work done by Foster.

The achieved results show that the speed of sound is a function of fluid and porous media characteristics and it is higher in rock that is more compact. Moreover, in the same porosity, the higher the permeability and fluid viscosity, the larger the sound speed. However, more experiments recommended for gaining more accurate conclusion.

In all the tests, the existance of Heavinside equation in Telegroph equation solution causes the pressure propagation front to specify like a jump in the profile.

The difference between two pressure equations shows that the difference is higher in early time but it becomes very small when time passes and in long cores in far distance from pressure pulse source.

By changing the rock and fluid parameters, it is noticeable that the correction of Telegraph equation is small comparing to the Darcy equation when the fluid has higher viscosity and for the rocks with low porosity. In the other word, however the Darcy equation has a physical limitation but its results is still reasonable in compare with Telegraph equation for conventional reservoir engineering applications.

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