The Heinemann-Mittermeir Generalized Shape Factor and Its Practical Relevance

Mohammad Taghi Amiry[*], Zoltan E. Heinemann and Clemens Brand
Leoben Mining University, Austria

Abstract

Fifty years ago Warren and Root have introduced the shape factor. This fundamental parameter for modeling of naturally fractured reservoirs has been discussed stormly ever since. Different definitions for shape factor have been suggested which all of them are heuristically based. Recently, Heinemann and Mittermeir mathematically derived - based on the dual-continuum theorem assuming pseudo-steady state condition- a general and proper form of the shape factor formula which can be simplified to the previously published shape factor definitions. This paper discusses the practical relevance of the Heinemann-Mittermeir formula. Its difference to the most commonly used Kazemi et al. formula is its demonstration by fine-scale single matrix block simulation. Furthermore, it is shown that the generally applied isotropy assumption can lead to significantly wrong results. Consequently, the generalized Heinemann-Mittermeir shape factor formula is recommended to be routinely practiced in the industry for more accurate results. The paper tries to present a proper realization of the nature of the shape factor as well as presentation of detailed mathematical and practical approaches for measuring all the required values in order to determine the shape factor for individual matrix rock pieces from outcrops of fractured formations. Performing those measurements routinely is regarded as essential parameter for its usability.

Introduction

The proper description of the recovery mechanisms of naturally fractured reservoirs is a challenging task. Barenblatt et al. [1] introduced the first dual-continuum concept which is widely used nowadays. In their model, most of the fluid is stored in matrix blocks of relatively low permeability, k_m, whereas the reservoir-scale permeability is due to an interconnected network of fractures. The fractures are not modeled explicitly, but are treated as a homogenized continuum, having the permeability of k_f. Fluid transfer between the fracture network and the matrix blocks at a given location is assumed to be proportional to the local difference between the fracture potential (Φ_f) and the average potential of the connected matrix blocks (Φ_m). Additionally, the volumetric fracture-matrix flux per unit volume of the fracture continuum is assumed to be proportional to the matrix permeability (k_m), inversely proportional to the fluid viscosity (μ) and to a parameter known as the shape factor (σ) as shown in Eq.(1):

$$q_{mf} = \frac{\sigma k_m}{\mu}\left(\Phi_f - \Phi_m\right) \qquad (1)$$

The shape factor has the unit of $[L^{-2}]$. Barenblatt et al. [1] did not discuss the physical meaning of the shape factor, other than to mention that it was inversely proportional to the square of some *"characteristic length"* of the matrix block. Since then, numerous equations for calculation of the shape factor have been proposed for various block shapes. The most commonly assumed shape has probably been a cube of length L. For cubical blocks, Warren and Root [2], based on an obscure derivation suggested the shape factor as $\sigma=60/L^2$. Kazemi et al. [3] suggested $\sigma=12/L^2$ for shape factor which is based on a finite-difference approximation to the flow equations with the entire matrix block represented by a single finite-difference cell. Coats [4] suggested the value $\sigma=24/L^2$ for shape factor calculation. Quintard and Whittaker [5] used "volume

averaging" to conclude that shape factor is equal to $49.62/L^2$.

Zimmerman et al. [6] used Fourier analysis with transient flow assumption and derived the shape factor as $\sigma=3\pi^2/L^2$. This value was confirmed by Lim and Aziz [13], and was re-derived by Mathias and Zimmerman [7] in the Laplace domain. Kazemi, Gilman and Elsharkawy (KGE) [8] suggested a generalized pseudo-steady state shape factor valid for all possible irregular matrix block shapes. Heinemann-Mittermeir [9] used a control volume finite-difference discretization on anisotropic dual continuum of irregular shape, and generalized the KGE [8] shape factor and mathematically proved that it is exact under pseudo-steady state condition for anisotropic matrix blocks as well.

The wide range of the values that have been proposed for the shape factor of a cubical block can in part, be explained by the fact that, as have been known for many decades (cf., de Swaan [10]), Eq.(1) is only accurate in the pseudo-steady state regime. Since Eq.(1) is not valid over all times, there is no unambiguous way to define the most appropriate value for σ. De Swaan [11] proposed that for the case of a step-change in the fracture pressure, σ can be chosen to render Eq.(1) accurately during the time in which the (mean) pressure change in the matrix block has attained 50% of its eventual value. This approach has some practical advantages, but causes the model to no longer be asymptotically accurate at large times.

In this paper, the most widely used shape factor after KGE [8] is studied for a matrix block of *anisotropic* permeability to demonstrate the effects of anisotropy on the matrix-fracture interaction. The output will be shown that this assumption can considerably affect on the results. Therefore, it is concluded that the anisotropy has to be taken into account for a proper forecast of the matrix-fracture interflow and is recommended that Heinemann-Mittermeir [9] shape factor which is a mathematically derived formula

which generalizes the KGE [8] shape factor for anisotropic rocks, should be used. Moreover, detailed mathematical methods to measure all the parameters required for calculation of the Heinemann-Mittermeir [9] shape factor of any piece of rock or for outcrops of fractured formations is described. Furthermore, a simplified more practical method for estimating these parameters from outcrops is presented that can be used in large-scale field studies.

Theory
Pseudo-steady state vs. transient

In small-scale laboratory experiments and measurements, the fluid transfer between the matrix and the fracture is affected by the transient period, causing different results from when considering pseudo-steady behavior. However, as discussed and stated by Sarma and Aziz [12]: "The shape factor converges asymptotically to the pseudo-steady state shape factor for dimensionless times greater than 0.1, which in typical reservoirs, the real time equivalent to this is usually very small *(e.g. in the scale of a few hours to just a few days).* Therefore it is often justified to only use the pseudo-steady state shape factor. The transient period can be significant in transient well tests, or in very tight gas reservoirs." This means that in practical field studies, where the time-steps are several days, the transient period would not have any effect on the result, and the pseudo-steady assumption accurately describes the conditions of the system (much less CPU-intensively). Therefore, in most of the industrial reservoir simulators, pseudo-steady state shape factors, such as KGE, is used. Moreover, as the term *"shape factor"* semantically suggests, it is a *factor* based on the *shape* (geometry) of the system. Therefore, it should necessarily be only dependent on the shape of the matrix block (and connectivity to the surrounding fractures).

However, some authors have considered the transient flow between matrix and fracture (which leads to shape factors that

change in time) and used the term *"time-dependent shape factor"* which is semantically incorrect as the *"shape"* of the system which does not change in time and therefore it is expected that the shape factor remains constant during the time too. The pseudo-steady state assumption, on the other hand, leads to a constant parameter for the geometry function i.e. *"shape factor"*.

It is noteworthy that the mentioned time-dependent shape factors are not meant to consider the effects of changes in the fracture network (e.g. changes in fracture connectivity); but they actually try to reflect the effects of transient state on the transfer function as a part of the shape factor. However, these effects actually should be accounted for the potential difference term which has the transfer function. Wherever they actually belong to, or can be ignored in full field cases, they would anyway converge asymptotically to the pseudo-steady stated values for larger time steps.

Permeability anisotropy

KGE [8] introduced a generalized shape factor for matrix blocks of any shape, with the assumption of pseudo-steady state and isotropic permeability. Heinemann and Mittermeir [9] mathematically derived the KGE shape factor formula and moreover generalized it also for anisotropic cases under pseudo-steady state.

Unlike the pseudo-steady state assumption (which does not have any practical effect in the full field studies), the isotropic-permeability assumption that many authors (cf. [1-4], [6-8], [10-11], [13]) have considered, can considerably affect the full field model behavior.

It should be mentioned that in this paper, the matrix block's permeability and its anisotropy is being discussed and the matrix simulation cells that are used in the simulation model is not being considered. In the other words, even in the dual-porosity single-permeability model (in which the matrix cells do not interact with their matrix neighbors), in order to transfer fluid to its fracture neighbor, a permeability value greater than zero needs to be assigned to the matrix block as the matrix permeability, k_m, in the definition of the conventional transfer function of Eq.(1). This permeability has to be specified for the matrix block to calculate the matrix-fracture transfer regardless the reservoir model (dual- or single-permeability). The above mentioned permeability is considered isotropic anyway in the conventional transfer function. To understand the differences more clearly, an example from what ECLIPSE [14] reservoir simulator done by default is explained: In a dual-permeability model, the transfer term (Eq.(1)) is calculated using the value of the x-direction permeability of the matrix cell, even if the matrix cell has different permeability values in other directions. The anisotropic permeability values would only be used when calculating the matrix-matrix transfer is desired, but for the matrix-fracture transfer term calculation, only the x-direction permeability value would be considered as if it were isotropic.

In this study, a method to measure the anisotropic permeability tensor on outcrops of reservoir formations is presented, and the effect of considering or ignoring the anisotropy in the reservoir behavior is discussed.

Shape vs. shape factor

The shape factor is generally based on the surface-to-volume ratio of the model as introduced by Barker [15]:

$$F_s = a^{-2}\alpha = \left(\frac{A_m}{V_m}\right)^2 \alpha \qquad (2)$$

where F_s is the shape function, a is the characteristic length, V_m is the volume, A_m is the surface of the matrix block and α is a dimensionless parameter. This means that the flow behavior of the model depends not really on the shape, but on its surface-to-volume ratio. Consequently, if the surface-to-volume ratios of two different matrix blocks of different shapes are the same, the matrix-fracture flow will be the same and as a result the driving forces will also the

same. The considered driving forces are capillary imbibitions (between the fracture and the matrix) and the gravity drainage (which depends on the height of the matrix block). Therefore, if the height of the blocks is the same (to impose the same gravitational drive), all models will have the same shape factor, as long as their surface-to-volume ratio is the same. This is illustrated in Figure 1 that how an arbitrarily shaped matrix block can have the same shape factor as a cuboids of the same height.
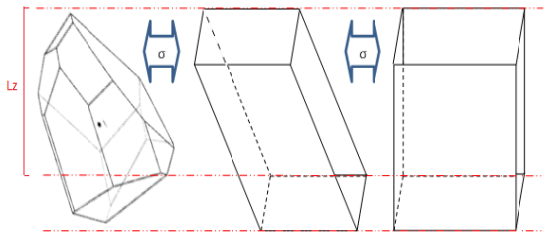

Figure 1: Different matrix block shapes with the same shape factor and flow behavior

This means that in order to model the flow behavior of a matrix block of any shape, it is possible to use a simple cuboids (which is easier to model and simulate) with the same "shape factor" as the irregularly shaped matrix block. The procedure is described later in the "Calculating the representative cuboids" section.

Calculating the shape factor

The pseudo-steady state shape factor can be calculated using KGE [8] formula:

$$\sigma^{KGE} k_m = k_m \frac{1}{V_m} \sum_{j=1}^{n} \frac{A_j}{d_j} \tag{3}$$

where V_m is the volume of the matrix block, A_j is the area of a surface open to flow and d_j is the distance from the open surface to the centroid of the matrix block.

Note that in KGE [8] shape factor, it is assumed that the matrix permeability (k_m) is isotropic and is the same in all directions.

The pseudo-steady state shape factor can be calculated using the Heinemann-Mittermeir (HM) formula [9], also taking the anisotropy into account:

$$\sigma^{HM} k_m = \frac{1}{V_m} \sum_{j=1}^{n} \frac{A_j}{d_j} \left| \overline{\overline{k}} \hat{n}_j \right| \tag{4}$$

where $\overline{\overline{k}}$ is the permeability tensor, and $\hat{n}$ is the unit normal vector of the surface open to flow in addition other parameters have been introduced previously in Eq.(3).

The parameters necessary to calculate the shape factor for an irregularly shaped piece of rock are described in the following sections.

Global coordinate system

Consider a homogeneous anisotropic matrix block of arbitrary shape. A global 3-dimensional Cartesian coordinate system is required to make the measurements. The origin can be any point in space (based on which all the distances will be measured) and the direction of the main axes can be also arbitrarily selected.
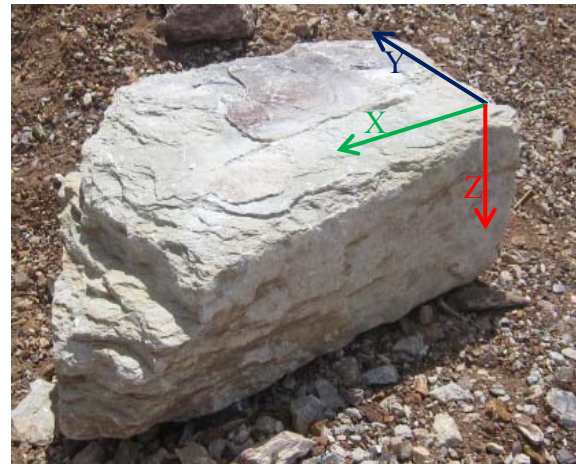

Figure 2: A real matrix block of irregular shape

Such an arbitrary global coordinate system and a real piece of matrix rock are shown in Figure 2.

Surface area measurement

Heron's formula, named after Heron of Alexandria, states that the T area of a triangle with side-lengths of a, b and c (as described by Dunham [16]) is:

$$T = \sqrt{s(s-a)(s-b)(s-c)}, \\ s = (a+b+c)/2 \tag{5}$$

This formula can be used to measure the surface of any irregular shape by masking it with non-overlapping triangles (as shown in Figure 3). Measuring their sides, calculating their areas and summing them up for all the

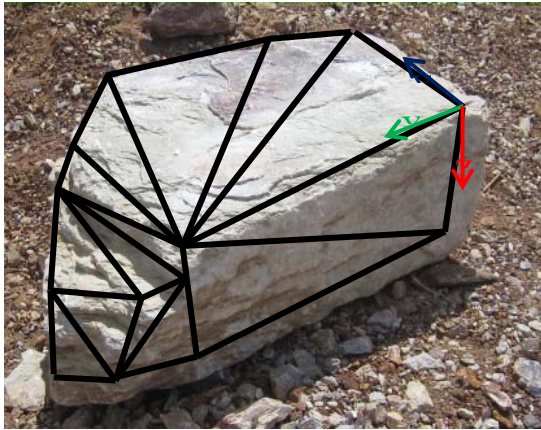triangles to calculate the surface area of the object.

Figure 3: Measuring the surface area and volume of an irregular shape

Measuring the volume of the block

The corner points of Figure 3 can be used to break the irregular shape to 3D-polygons (such as pyramids) and easily calculate the total volume of the block (V_m) by summing the volumes of these polygons up.

Finding the centroid of the block

The centroid point of the matrix block shape can be calculated from the positions of the corner points (in Figure 3):

$$\vec{r}_c = \frac{1}{n}\sum_i^n \vec{r}_i \tag{6}$$

where $\vec{r}_c$ is the position vector of the centroid, n is the total number of corner points and $\vec{r}_i$ is the position vector of each corner point of the block.

Finding the unit normal vectors of each surface of the block

Three arbitrary points are chosen on the desired surface: $O(o_x, o_y, o_z)$, $A(a_x, a_y, a_z)$ and $B(b_x, b_y, b_z)$ as shown in Figure 4. The unit normal vector $\hat{n}$ can be calculated from the cross product vector of the two vectors $\overrightarrow{OA}(a_x - o_x, a_y - o_y, a_z - o_z)$ and $\overrightarrow{OB}(b_x - o_x, b_y - o_y, b_z - o_z)$ divided by the length of the product as shown in Eq.(7):

$$\hat{n} = \frac{\overrightarrow{OA} \times \overrightarrow{OB}}{\left\|\overrightarrow{OA} \times \overrightarrow{OB}\right\|} \tag{7}$$
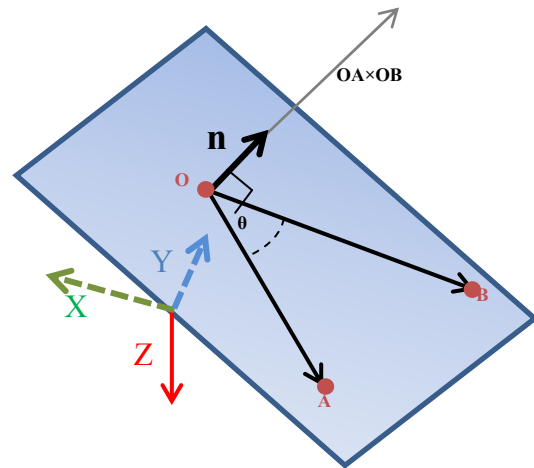

Figure 4: Finding the unit normal vector of the surface

Surface distance from the centroid

The distance of a point to a surface can be calculated from Eq.(8) (Wolfram MathWorld [17]):

$$d = \hat{n} \cdot (\vec{r}_O - \vec{r}_c) \tag{8}$$

Where d is the distance of the surface to the centroid, $\hat{n}$ is the unit normal vector of the plane, $\vec{r}_O$ is the position vector of any point on the surface and $\vec{r}_c$ is the centroid's position vector.

Permeability tensor measurement

In this section, it is described how to find the principle permeability direction which yields the coordinate system in which the permeability anisotropy tensor is diagonal. Measuring the permeability tensor is quite a challenging task which many authors such as Lishman [18], Mousatov et al. [19], Durlofsky [20], Rose [21], Walter [22] or Weitzenböck et al. [23] have tried to propose different methods or to design instruments to measure it.

In this paper a method which was suggested by Asadi et al. [24] is briefly explained. A sample from the rock that is large enough to eliminate the end-effect is shaped as described by Asadi et al. [24] (which is the 3D extension of Rose [21] method for 2D):
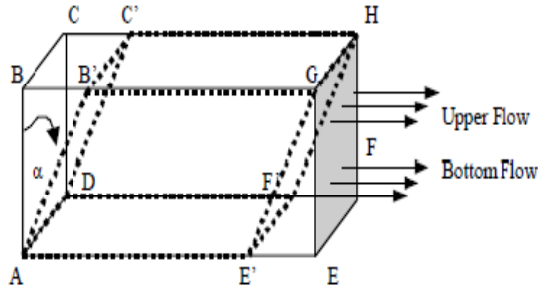
Figure 5: The first step for reshaping the cubic sample

The cubic sample is reshaped into a parallelepiped in three steps performed in each direction one after another. The angle of each outlet face of the sample is changed so that the upper flow and the lower flow become the same (and the inlet face angle is also changed in parallel to the outlet face).

Figure 5 shows the first step of reshaping the cube sample. The same procedure is also performed on the other two directions and ultimately the block is reshaped into a parallelepiped as shown in Figure 6. The plot of angle vs. $\ln\left(\dfrac{q_{top}}{q_{bottom}}\right)$ shall produce the final values for the angles of each face.
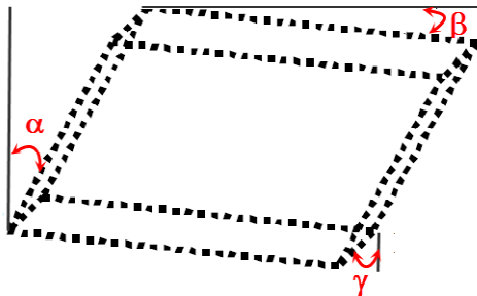

Figure 6: The final parallelepiped formed by reshaping the cubic sample

It is expected that the direction of pressure gradient along each side of this parallelepiped is parallel to the flow [24], [21]. Now three permeability measurement experiments can be performed along each side of the parallelepiped to measure the pressure drop for a given flow rate and therefore the Darcy's flow equation is used for that face as in Eq.(9):

$$(9)$$

$$\vec{Q} = \frac{-\bar{\bar{k}}}{\mu}\nabla\vec{P} \Rightarrow$$

$$\begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \frac{-1}{\mu}\begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}\begin{bmatrix} \partial p/\partial x \\ \partial p/\partial y \\ \partial p/\partial z \end{bmatrix}$$

where the 9 permeability tensor values are unknown. Three additional equations come from the fact that the permeability tensor is symmetric by the Onsager [25] reciprocal relations. This means that:

$$(10)$$

$$k_{ij} = k_{ji}, \quad i,j \in \{x,y,z\}.$$

In order to calculate the remaining unknowns, Asadi et al. [24] consider the directions of maximum, minimum and intermediate permeability and their angles to the direction of pressure gradient: The angles of the maximum permeability direction to each component of the pressure gradient vector are M_1, M_2, M_3 respectively and for the minimum permeability direction are m_1, m_2, m_3 and for the intermediate permeability direction (which is perpendicular to both minimum and maximum permeability directions) are i_1, i_2, i_3.

The principal permeability k_{xx}, k_{yy}, k_{zz} of the nine element symmetric tensor are the eigenvalues of the following matrix (Anton and Rorres [26]):

$$(11)$$

$$\begin{bmatrix} k_\lambda^{-1} & k_{xy} & k_{xz} \\ k_{yx} & k_\lambda^{-1} & k_{yz} \\ k_{zx} & k_{zy} & k_\lambda^{-1} \end{bmatrix} = 0$$

Eq.(11) is cubic in λ and has three real roots which correspond to the three values of k_{max}, k_{int} and k_{min}. To solve Eq.(11) for the principle permeability, the principle axis are written to satisfy the following equations for the directional cosines of the principal [24]:

$$\begin{bmatrix} k_\lambda^{-1} & k_{xy} & k_{xz} \\ k_{yx} & k_\lambda^{-1} & k_{yz} \\ k_{zx} & k_{zy} & k_\lambda^{-1} \end{bmatrix}\begin{bmatrix} \cos M_1 \\ \cos M_2 \\ \cos M_3 \end{bmatrix} = 0 \quad (12)$$

$$\begin{bmatrix} k_\lambda^{-1} & k_{xy} & k_{xz} \\ k_{yx} & k_\lambda^{-1} & k_{yz} \\ k_{zx} & k_{zy} & k_\lambda^{-1} \end{bmatrix}\begin{bmatrix} \cos m_1 \\ \cos m_2 \\ \cos m_3 \end{bmatrix} = 0 \quad (13)$$

$$\begin{bmatrix} k_\lambda^{-1} & k_{xy} & k_{xz} \\ k_{yx} & k_\lambda^{-1} & k_{yz} \\ k_{zx} & k_{zy} & k_\lambda^{-1} \end{bmatrix}\begin{bmatrix} \cos i_1 \\ \cos i_2 \\ \cos i_3 \end{bmatrix} = 0 \quad (14)$$

$$\quad (15)$$

$$\cos^2 M_1 + \cos^2 M_2 + \cos^2 M_3 = 1$$
$$\cos^2 m_1 + \cos^2 m_2 + \cos^2 m_3 = 1$$
$$\cos^2 i_1 + \cos^2 i_2 + \cos^2 i_3 = 1$$

Eqs.(9-(15) altogether contain 18 unknowns and 18 equations and by solving them, all the permeability tensor elements can be calculated as explained by Asadi et al. [24].

Principle permeability coordinate system and permeability tensor diagonalization

Since the permeability tensor is[1] both symmetric and positive definite[2], it is always diagonalizable [26].

In order to diagonalize the permeability tensor (obtained from any method), its eigenvectors of $\vec{X}_1, \vec{X}_2, \vec{X}_3$, respectively for the eigenvalues of k_x, k_y and k_z should be calculated. Eq.(16) is the definition of eigenvalues and eigenvectors for the permeability tensor:

$$\overline{\overline{k'}}\vec{X}_1 = k_x\vec{X}_1,$$
$$\overline{\overline{k'}}\vec{X}_2 = k_y\vec{X}_2,$$
$$\overline{\overline{k'}}\vec{X}_3 = k_z\vec{X}_3 \quad (16)$$

$$E = [\vec{X}_1, \vec{X}_2, \vec{X}_3] = \begin{bmatrix} X_{1_x} & X_{2_x} & X_{3_x} \\ X_{1_y} & X_{2_y} & X_{3_y} \\ X_{1_z} & X_{2_z} & X_{3_z} \end{bmatrix}$$

where E is the matrix comprised of eigenvectors of $\overline{\overline{k'}}$ written as column vectors and put side by side.

As mentioned, $\overline{\overline{k'}}$ is positive definite, therefore E is an orthonormal[3] matrix and represents simply a rotation transform (Anton and Rorres [26]). Since k_x, k_y, k_z are the eigenvalues of the permeability tensor, Eq.(17) and as a result, Eq.(18) are always correct by definition:

$$\overline{\overline{k'}}E = E.\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix} \Rightarrow \quad (17)$$

$$\overline{\overline{k'}} = E.\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}.E^{-1} \Rightarrow$$

$$\overline{\overline{k'}} = E\overline{\overline{k}}E^{-1} \quad (18)$$

where $\overline{\overline{k}} = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}$ is the diagonalized permeability tensor.

This expression helps describe the procedure of matrix diagonalization in the following sequence of operations (reading Eq.(18) from the right):

1. At first multiplying by E^{-1}. Think of this as performing a linear change of the coordinates, i.e. changing the coordinates to some special coordinate system called the "*principle permeability coordinate system*".

2. As the Second step multiplying by $\overline{\overline{k}}$. But this is a particularly easy matrix to multiply with, since the coordinates do not mix: it means that each coordinate gets stretched or squeezed by just one of the eigenvalues of $\overline{\overline{k'}}$.

3. Finally, going back to the global coordinates, multiplying by the inverse of E^{-1}, namely E.

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[1] http://en.wikipedia.org/wiki/Permeability_(earth_sciences)

[2] Positive definite matrix: a matrix **M** such that $\mathbf{x}^T\mathbf{Mx} > 0$ for all non-zero **x**.

[3] Orthonormal matrix: matrix **M** where $\mathbf{M}^T\mathbf{M} = \mathbf{I}$.

Note that the order of choosing the eigenvalues as k_x, k_y or k_z is not important, since the rotation matrix is created by the corresponding eigenvectors in the same order as the selected eigenvalues. In the other words, the values of the diagonalized permeability tensor in the transformed coordinate system will be always the same, regardless of the order of selection of the eigenvalues.
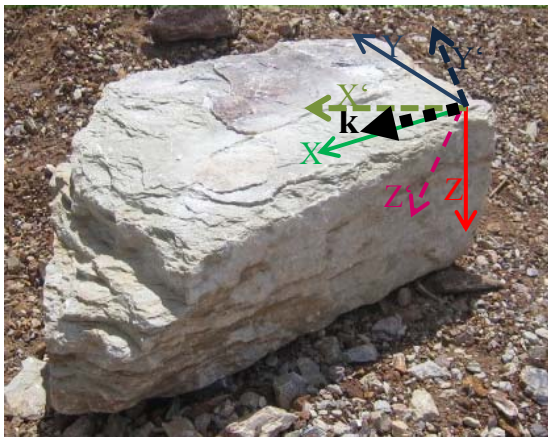

Figure 7: Principle permeability coordinate system

Applying the rotation transform of E^{-1} to the global coordinate system, the principle permeability coordinate system (X'-Y'-Z') can be calculated as shown in Figure 7 with the dashed arrows. In this coordinate system, the diagonalized permeability tensor can be directly used in calculations.

Equivalent isotropic permeability and normalized anisotropy tensor

Muskat [27] introduced the term *"equivalent isotropic permeability"* to calculate the isotropic permeability in case of having anisotropic permeability tensor (to be used in calculations and simulations that are based on the isotropy assumption). This term is nothing more than the geometric mean of the directional permeability values as shown in Eq.(19):

$$k_m = \sqrt[3]{k_x k_y k_z} \qquad (19)$$

This value can be used to normalize the permeability tensor to a dimensionless tensor that represents the anisotropy of the medium which is called *"normalized anisotropy tensor"*, $\bar{\bar{k}}$, which represents the anisotropy of the rock:

$$\bar{\bar{k}} = \bar{\bar{k}} / k_m = \begin{bmatrix} k_x/k_m & 0 & 0 \\ 0 & k_y/k_m & 0 \\ 0 & 0 & k_z/k_m \end{bmatrix} \qquad (20)$$

Permeability and shape factor in the transfer term

As shown in Eq.(1), the dimension of the shape factor is [m^{-2}] which does not consider the permeability. Therefore, sometimes instead of considering the shape factor and permeability separately, the term σk with the dimension of [D/m^2] (Darcy per square meter) is considered as a single entity in the transfer term to be able to have a permeability-dependent term as mentioned in the Eq.(4).

Calculating the representative cuboid

The representative cuboids which has the same shape factor as the irregularly shaped block can be calculated assuming $l_x = l_y$, and l_z = *the mean height of the original block* (to keep the effects of gravity, as the height-dependent parameter, the same between the cuboid and the original rock). The values can be easily calculated for a cuboid from Eq.(3) for KGE shape factor or Eq.(4) for Heinemann-Mittermeir shape factor:

$$l_x^{KGE} = l_y^{KGE} = \sqrt{\frac{2}{\dfrac{\sigma^{KGE}}{4} - \dfrac{1}{l_z^2}}} \qquad (21)$$

$$l_x^{HM} = l_y^{HM} = \sqrt{\frac{k_x + k_y}{\dfrac{\sigma^{HM}}{4} - \dfrac{k_z}{l_z^2}}} \qquad (22)$$

Studying the effect of permeability anisotropy
The numerical model

In order to study the flow behavior of the matrix block, a mathematical model was created to simulate the laboratory experiment of submerging a single matrix

block in a fluid (either water or gas) under different conditions.

The model inputs the shape factor and internally sets up the representative cuboid (which has the same flow behavior as the irregularly shaped matrix block) using either Eq.(21) or Eq.(22) as desired. The matrix block (representative cuboid) is then discretized to small simulation *cells* (forming the *matrix domain*, red cells in Figure 8) which are surrounded by fracture *cells* (forming the *fracture domain*, green cells in Figure 8). Both domains are treated as two single-porosity volume-regions that can interact with each other in the conventional single-porosity manner.

The only difference between the two domains is that the matrix domain has the endpoints and rock properties of the matrix domain and the fracture domain are those of the fracture domain (i.e. straight-line relative permeabilities and zero capillary pressure). Only the fluid content of the matrix domain is of interest and the fracture domain acts only as a constant-pressure boundary with constant fluid content: either is fully gas-filled or is fully water-filled to represent submerging the matrix block in gas or water respectively. The matrix domain initially has the maximum oil saturation.
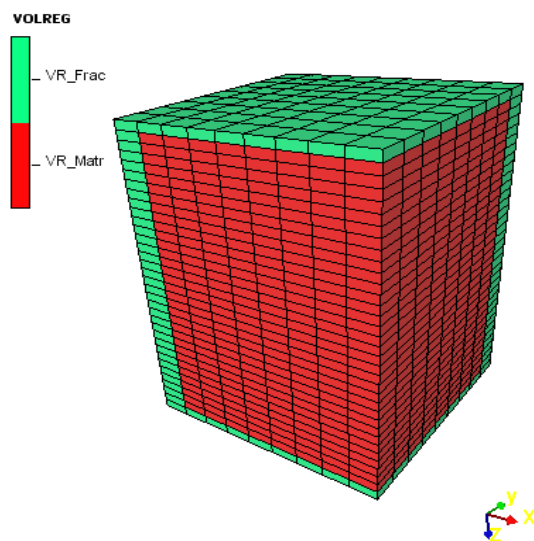

Figure 8: Single Matrix Block model (a quarter of the actual matrix block)

The amount of oil that flows out of the matrix domain to the fracture domain is collected at the end of each time-step and after being divided by the matrix domain initial oil volume, is reported as the recovery factor for that time-step. The entered oil to the fracture domain is taken out of the system, to keep the fracture saturation constant in time.

Since the matrix block is considered to be homogeneous, the flow will be symmetrical and it will be enough that the calculations are performed on a quarter of the model as shown in Figure 8 and assigned to the other three quarters as well. This makes the run about four times more efficient (which is very beneficial since the time-steps may need to be small due to the small scale of the model).

The test cases

In order to demonstrate the effect of permeability anisotropy on matrix-fracture interaction, an irregularly shaped piece of rock (assuming bulk homogeneity) is modeled and its shape factor is calculated using KGE formula (Eq.(3)) and the representative cuboid is calculated (using Eq.(21)).

The measured shape factor of this piece of rock and its calculated representative cuboid size are as follows:

$$\sigma^{KGE} = 0.183 / m^2, \quad l_z = 7.62m, \quad \Rightarrow$$

$$l_x^{KGE} = l_y^{KGE} = 6.61m$$

Two cases are considered with different permeability anisotropy tensors: The permeability tensor for Case 1 is:

$$k_1 = \begin{bmatrix} 4.5 & 0 & 0 \\ 0 & 0.12 & 0 \\ 0 & 0 & 0.12 \end{bmatrix} mD \Rightarrow k_{m1} = 0.4mD$$

And the permeability tensor for Case 2 is:

$$k_2 = \begin{bmatrix} 2.0 & 0 & 0 \\ 0 & 2.0 & 0 \\ 0 & 0 & 0.25 \end{bmatrix} mD \Rightarrow k_{m2} = 1.0mD$$

The (discretized) matrix cells are filled with maximum oil saturation, and the fracture cells act as a boundary condition containing always 100% water. Due to capillary pressure and gravity drainage, the water enters the matrix cells and expels the oil out to the fracture cells. The amount of oil outflow of the matrix in each time-step is collected from the fracture cells and after dividing it by the initial matrix oil content, will be reported as the recovery factor in that time-step.

For each permeability tensor, two scenarios for the permeability are assumed (while all other parameters remain identical):

1. The permeability in all directions are set to the calculated equivalent isotropic permeability, k_m.
2. The diagonalized anisotropic permeability, $\overline{\overline{k}}$, is used to have different permeability values in each direction (i.e. the principle permeability coordinate system is assigned as the model coordinate system).

Figure 9 illustrates the results (time vs. recovery factor) of the two scenarios for both cases: Case 1 is plotted in blue and Case 2 is plotted in red. For each case, the isotropic permeability is plotted with the dashed line and the anisotropic permeability with the solid line.
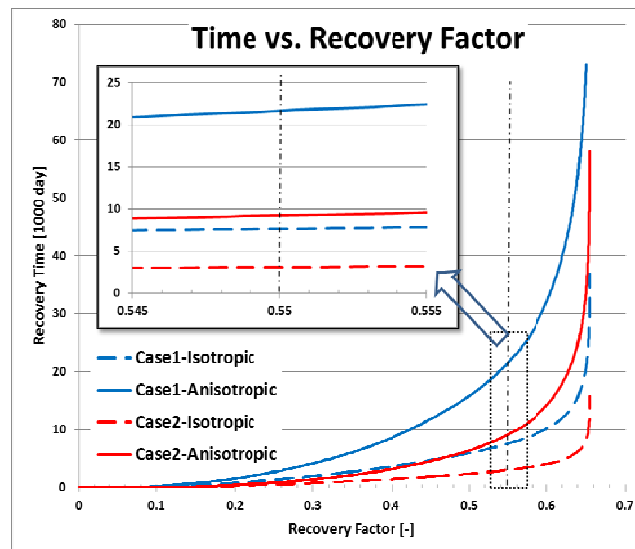

Figure 9: Time of recovery vs. Recovery factor for four permeability scenarios
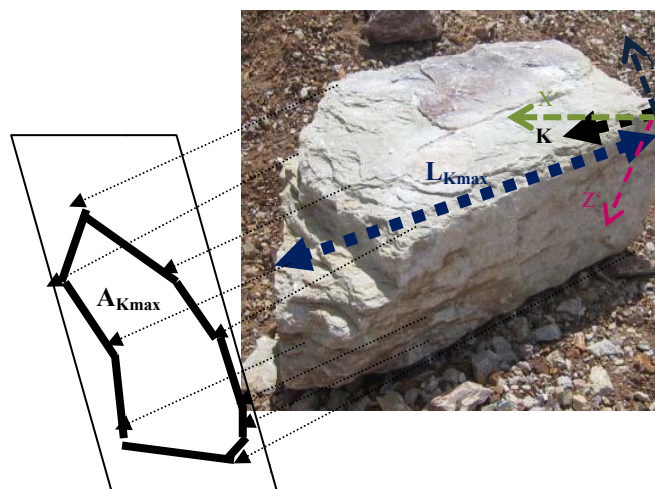

Figure 10: Area and length of the matrix block in the maximum permeability direction

As can be observed, each scenario presents a completely different trend of recovery: 55% recovery of the matrix oil content for Case 1, takes nearly 8000 days (~22 years) with isotropy assumption while it takes about 21000 days (~58 years) considering the anisotropy. For Case 2 it takes about 3000 days (~8 years) with isotropy assumption and about 9000 days (~24 years) with anisotropy assumption.

The difference in recovery time of 55% between the isotropic and anisotropic scenarios for Case 1 is ~36 years and for Case 2 is ~16 years which are quite remarkable differences in estimation of the recovery time from a small matrix block, with the same initial and boundary conditions only as a result of considering or ignoring the anisotropy.

It is also observable that all the curves start at the same obvious recovery value of zero (while the initial conditions are the same) but also end at the same common recovery value of 64.68% as the ultimate recovery. The reason is that the ultimate recovery from the matrix block only depends on the driving mechanisms in action and the saturation endpoints, which are the same in all scenarios and are not dependent on the permeability. In the other words, all scenarios will ultimately produce the same amount of oil but with different trends and time scale. However, it is essential that this trend is known as accurate as possible to be able to make appropriate plans for the production from naturally fractured reservoirs.

Practical simplifications

The presented method to measure the shape factor parameters from outcrops, is mathematically accurate, but may be considered too difficult to manually practice on a large scale/quantity.

However, there are software programs that help make 3-dimensional models from the pieces of rock just by analyzing the pictures in different angles from the rock (such as 3D Software Object Modeller Pro [28]). These 3D models can be then used to

easily determine the surface area, centroid and the distance of each face to the centroid and to ultimately calculate the shape factor from Eq.(4).

An alternative practical method is to express the Heinemann-Mittermeir [9] shape factor formula in an approximated form to facilitate intuitively estimation of the parameters as shown in Eq. (23):

$$\sigma^{HM} \approx \frac{1}{V_m}\left(\frac{A_{k\,min}}{d_{k\,min}} + \frac{A_{k\,max}}{d_{k\,max}} + \frac{A_{k\,int}}{d_{k\,int}} \right) \qquad (23)$$

where V_m is the volume of the matrix block, A_K is the projected area of the matrix block on a plane orthogonal to the maximum, minimum or intermediate permeability directions and d_K is half of the mean length of the matrix block (L_K) in that direction. Figure 10 shows the schematic representation of A_{Kmax}, the projected area and L_{Kmax}, the mean length of the matrix block in the direction of maximum permeability.

It is possible to estimate A_K and L_K as well as the matrix block's volume, V_m, intuitively after a few measurements, which makes onsite utilizing of this shape factor formula more practical. The principle permeability *direction* may be estimated itself by close inspection of the formation sedimentation.

The permeability *values* can be measured in lab for some cores and averaged for the formation.

Note that in Eq. (23), the *normalized* directional permeability values are not explicitly observable; as their effect is already included in the term A_K/d_K (i.e. since this term is determined in the principle permeability directions, they are *dependent* on the anisotropic permeability tensor).

In Eq. (23), the term σ^{HM} is calculated rather than $\sigma^{HM}k_m$ as in Eq.(4), and therefore, in order to calculate the transfer term, the average permeability k_m has to be multiplied (by the shape factor value) as well; however, as already mentioned, using the equivalent isotropic permeability here, does not undermine the matrix anisotropy, since the shape factor is already calculated

considering the *normalized* permeability tensor i.e. in the principle permeability direction.

Such simplifications make it practical to use field outcrop investigations to measure the anisotropic shape factor more accurately than what is currently being used, but still not as time-consuming and as accurate as measuring all the parameters in the lab for every piece of rock, as was described earlier in this paper (which makes it impractical on a large scale).

Conclusions

This study, using four different scenarios which were identical except for permeability tensor while follow completely different trends of production, demonstrates that although permeability anisotropy is not considered routinely in petroleum engineering studies, it can play a significant role in depletion trend of matrix blocks in naturally fractured reservoirs.

Therefore, use of a shape factor such as Heinemann-Mittermeir which considers the anisotropy, also is theoretically derived and mathematically proven, is highly recommended rather than using the simplified, commonly used Kazemi et al. isotropic shape factor which can result in quite wrong estimations of matrix depletion trend.

Mathematical methods to determine all the parameters of Heinemann-Mittermeir shape factor for the matrix blocks of any shape (from the outcrops of fractured formations or elsewhere) were described and additionally, a possible simplified and more practical approach to intuitive estimation of those parameters was presented. It is suggested that field outcrop investigations should be performed for measuring the shape factor more accurate for the naturally fractured formations using the presented methods (especially in cases of high anisotropy).

The test cases also suggest that using the apparent permeability in the transfer function which ignores the effect of anisotropy yet in a higher level than the shape factor, can be a more serious source of discrepancies in the matrix-fracture transfer rate calculation as well.

References:

1- Barenblatt, G. J., Zheltov, I. P. and Kochina, I. N. (1960). "Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks." *J. Appl. Math. Mech.*, Vol. 24, pp. 1286-1303.

2- Warren, J.E. and Root, P.J. (1963). The Behavior of Naturally Fractured Reservoirs, SPE Journal, (Sept. 1963), 245-255.

3- Kazemi, H., Merrill, L.S., Porterfield, K.L. and Zeman, P.R. (1976). Numerical Simulation of Water-Oil Flow in Naturally Fractured Reservoirs, SPE Journal, 317-326.

4- Coats, K.H. (1989). Implicit Compositional Simulation of Single-Porosity and Dual-Porosity Reservoirs. SPE paper 18427.

5- Quintard, M. and Whitaker, S. (1996). "Theoretical development of region-averaged equations for slightly compressible single-phase flow." *Adv. Water Res.*, Vol. 19(1), pp. 29-47.

6- Zimmerman, R.W., Chen, G., Hadgu, T. and Bodvarsson, G.S. (1993). "A numerical dual-porosity model with semi-analytical treatment of fracture/matrix flow." *Wat. Resour. Res.*, Vol. 29(7), pp. 2127-2137.

7- Mathias, S.A. and Zimmerman, R.W. (2003). Laplace transform inversion for late-time behavior of groundwater flow problems. Water Resources Research, 39(10): paper 1283.

8- Kazemi, H., Gilman, J.R. and Elsharkawy, A.M. (1992). Analytical and Numerical Solution of Oil Recovery from Fractured Reservoirs Using Empirical Transfer Functions, SPE Reservoir Engineering, May 1992, 219-227.

9- Heinemann, Z.E. and Mittermeir, G.M. (2012). "Derivation of the Kazemi-Gilman-Elsharkawy generalized dual porosity shape factor." *Transp. Porous Med.*, Vol. 91(1), pp. 123-132.

10- de Swaan, A. (1976). Analytic Solutions for Determining Naturally Fracture Reservoir Properties by Well Testing. SPE Paper, 5346-PA.

11- de Swaan, A. (1990). Influence of Shape and Skin of Matrix-Rock Blocks on Pressure Transients in Fractured Reservoirs. SPE Formation Evaluation, Dec. 1990, 344-352.

12- Sarma, P. and Aziz, K. (2006). Production Optimization With Adjoint Models Under Nonlinear Control-State Path Inequality Constraints. SPE Paper, 99959-MS.

13- Lim, KT. And Aziz, K. (1995). "Matrix-fracture transfer shape factors for dual-porosity simulators." *J. Pet. Sci. Eng*, Vol. 13, pp. 169-178.

14- Schlumberger, (2012). ECLIPSE reservoir simulation software – Technical Description Version 2012.1.

15- Barker, J.A. (1985). "Block-geometry functions characterizing transport in densely fissured media." *J. Hydrol.*, Vol. 77, pp. 263-279.

16- Dunham, W. (1990). Heron's Formula for Triangular Area. Ch. 5 in Journey through Genius: The Great Theorems of Mathematics. New York: Wiley, pp. 113-132.

17- Wolfram Math World, http://mathworld.wolfram.com/Point-PlaneDistance.html

18- Lishman, J.R. (1970). Core Permeability Anisotropy, *J. Canadian Pet. Tech.*, 9(2).

19- Mousatov, A., Pervago, E. and Shevnin, V. (2000). A New Approach to Resistivity Anisotropy Measurements, SEG 2000 Expanded Abstracts, 2000-1381.

20- Durlofsky, L.J. (1991). "Numerical calculation of equivalent grid block permeability tensors for heterogeneous porous media." *Wat. Resou. Res.*, Vol. 27(5), pp. 699-708.

21- Rose, W. (1982). "A new method to measure directional permeability." *J. Petrol. Technol.*, May 1982.

22- Walter D., R. (1982). "A new method to measure directional permeability." *J. Petrol. Eng.,* Vol. 34(5), pp. 1142-1144.

23- Weitzenböck, J.R., Shenoi, R.A. and Wilson, P.A. (1997). "Measurement of three-dimensional permeability." *Appl. Sci. Manuf.*, Vol. 29, pp. 159-169.

24- Asadi, M., Ghalambor, A., Rose, W.D. and Shirazi, M. K. (2000). Anisotropic Permeability Measurement of Porous Media: A 3-Dimensional Method. SPE Conference Paper, 59396.

25- Onsager, L. (1931). "Reciprocal relations in irreversible processes." *Phys. Rev.* Vol. 37, pp. 405-426.

26- Anton, H. and Rorres, C. (2000). Elementary Linear Algebra (Applications Version) 8th edition, John Wiley & Sons. ISBN 978-0-471-17052-5.

27- Muskat, M. (1937). The flow of homogeneous fluids through porous media, Mc Graw Hill Book Company Inc., NY, USA, 137-148.

28- Creative Dimension Software Ltd – 3D Software Modeller Pro.