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Investigation of Mass Transfer Diffusivity Dependency in Drying Process of Lemon

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ABSTRACT

Achieving the rate and the amount of mass transfer is of paramount importance in selecting optimum conditions for drying and affects the development of the quality of drying. Note that, to obtain the amount of mass transfer, the conditions of mass transfer such as temperature, pressure, geometry, and diffusion coefficient should be completely determined. In this research, an experiment is conducted in atmospheric conditions and then the amount of mass flow in a spherical body is measured. Utilizing the Newman equation and the experimental results, the diffusion coefficient is found to be in the range of 10^{-11} m^2/s. Additionally, the experimental data reveal the linear and exponential variation of diffusion coefficient with a constant coefficient of 1306.8 and exponent of 2.0883 which is against size and time.

Introduction

The drying of fruits and vegetables has an important role in the food industry. Studying the way that mass transfer and heat transfer occur, has been an interesting subject for researchers for improving the quality of the goods and the efficiency of the process [1- 8]. For example, Sahin et al. [9] studied the effects of temperature and humidity of the air on the rate of mass and heat transfer. Bala et al. [10] investigated a sunshine drying process and drying system with a hot air tunnel; and realized that tunnel with hot air needs less time for the same drying process. Sopian et al. [11] studied a drying system with both a hot air fan and sunshine. Jain et al. [12] realized that the rate of mass transfer in the drying process is a function of internal and external effects like sunshine energy, the surrounding temperature, air velocity, humidity, initial concentration, and the kind of fruit or vegetable. Also, Tiris et al. [13] investigated a drying system with fan control and found similar results. Furthermore, sunshine had been used as a renewable source of energy for the drying process. Bena et al. [14] completed the natural

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convection of sunshine by a biomass fuel and studied the mass transfer of fruits and vegetables on a small scale. Although researchers have studied different drying systems with forced and natural convection, studies on drying methods continue. Finding the diffusivity of the drying process has always been important for researchers; because knowing how diffusivity changes with temperature, time and superficial properties is necessary for choosing the best conditions for the drying process. Conway et al. [15] determined the diffusivity of potatoes as (7.663 - 35.93)×10^{-11} m^2/s and Mosayebi et al. [16] detected diffusivity as (3.45 - 4.39)×10^{-9} m^2/s. Also, Sadeghi et al. [17] studied the optimization of the lemon drying process and reported the diffusivity of lemon slices as (1.2 – 2.4) × 10^{-9} m^2/s. Studying diffusivity in the drying process can give us a lot of information about the mass transfer, amount of diffusivity, and mass transfer location. In this research, by assuming the diffusivity is independent of natural or forced convection, the simplest way by using sunshine is used. Then, the Newman equation is used for mass transfer in unsteady solids to find the effective diffusivity of lemon in lemon body by the data got from the experiment. Finally, the effective diffusivity of lemon as a function of time and lemon diameter to optimize the drying process are chosen.

Theory

Mass transfer is studied in steady and unsteady states by using Fick's first and second laws [2]. In steady-state conditions, Fick's first law says the rate of mass transfer dependency with concentration gradient:

$$J = -D_{AB}\frac{dC_A}{dz} \tag{1}$$

The constant in this equation is called diffusion coefficient or diffusivity.

In unsteady states using mass balance is as below:

$$\dot{m}_{A_1} - \dot{m}_{A_2} + R_A M_A \Delta V = \frac{dm_A}{dt} \tag{2}$$

Then by assuming lemon as a sphere the result is:

$$\frac{\partial C_A}{\partial t} + \left(\frac{1}{r^2}\frac{\partial C_A}{\partial t}(r^2 N_{Ar}) + \frac{1}{rsin\theta}\frac{\partial}{\partial\theta}(N_{A\theta}sin\theta) + \frac{1}{rsin\theta}\frac{\partial N_{A\emptyset}}{\partial\emptyset}\right) = R_A \tag{3}$$

By assuming constant density and diffusivity, the result is:

$$\frac{\partial C_A}{\partial t} + \left(u_r\frac{\partial C_A}{\partial r} + u_\theta\frac{1}{r}\frac{\partial C_A}{\partial\theta} + u_\emptyset\frac{1}{rsin\theta}\frac{\partial C_A}{\partial\emptyset}\right) = D_{AB}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial C_A}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 C_A}{\partial\emptyset^2}\right) + R_A \tag{4}$$

When there is no chemical reaction and bulk motion (we can ignore the internal motions of lemon like a solid sphere) and by assuming mass transfer in radius direction; it results:

$$\frac{\partial C_A}{\partial t} = D_{AB}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_A}{\partial r}\right)\right) \tag{5}$$

Then:

$$\frac{\partial C_A}{\partial t} = D_{AB}\left(\frac{2}{r}\frac{\partial C_A}{\partial r} + \frac{\partial^2 C_A}{\partial r^2}\right) \tag{6}$$

It shows that the mass transfer mechanism in lemon is diffusion. The boundary and initial condition for solving differential Eq. 6 are discussed as follows. Solute concentration at start time in all spots of the sphere is C_{A0} and initial conditions are as follows:

$$r = r\ ;\ 0 \le r \le r_s \quad ; \quad t = 0 \ \Rightarrow \ C_A = C_{A_0} or\ \ C_A(r,0) = C_{A_0}$$

By ignoring resistance in the continuous phase, the concentration of A on the sphere interface is C_{A^*} (equilibrium concentration with continuous phase):

$$BC_1 : t = t\ ;\ r = r_s \Rightarrow \quad C_A = C_A^* \quad or \quad C_A(r_s\ ,t) = \ C_{A^*}$$

Assuming the mass transfer only in *r* direction and constant physical properties (because of the low rate of mass transfer) results that the concentration in the center of the sphere is limited.

$$BC_2 : \lim C_A(r\ ,t) = limited$$

By using the separation of variables method for solving a partial differential equation, it will be solved as bellows:

$$y(r,t) = \ C_A - C_A^*$$

$$BC_1 : y(r_s\ ,t) = 0$$

$$BC_2 : \ \lim y(r\ ,t) = 0$$

$$IC : \ y(r\ ,0) = \ C_{A_0} - C_A^*$$

Using method of separation of variables:

$$y(r\ ,t) = R(r)T(\theta)$$

$$R(r) = \frac{C_2}{r}\sin\left(\frac{n\pi r}{r_s}\right) \quad , \quad n = 1,2,3,\ldots \tag{7}$$

$$T(\theta) = C_1 Exp(-\lambda^2 Dt) \quad , \quad \lambda = \frac{n\pi}{r_s} \tag{8}$$

Then;

$$y = \sum_{n=1}^{\infty} A_n \frac{1}{r}\sin\left(\frac{n\pi r}{r_s}\right) Exp\left(-\frac{Dn^2\pi^2 t}{{r_s}^2}\right) \tag{9}$$

By applying on Fourier's series result in:

$$f(r) = r\left(C_{A_0} - C_A^*\right) \ ,$$

$$A_n = \frac{2}{r}\int_0^{r_s} r\left(C_{A_0} - C_A^*\right) \sin\left(\frac{n\pi r}{r_s}\right) dr = \frac{2r_s}{n\pi}\left(C_{A_0} - C_A^*\right)(-1)^{n+1} \tag{11}$$

$$r\left(C_{A_0} - C_A^*\right) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi r}{r_s}\right) \tag{10}$$

Then;

$$\left(C_A - C_A^*\right) = \frac{2r_s}{\pi}\left(C_{A_0} - C_A^*\right)\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\frac{1}{r}\sin\left(\frac{n\pi r}{r_s}\right) Exp\left(-\frac{Dn^2\pi^2 t}{{r_s}^2}\right) \tag{12}$$

Eq. 12 shows the concentration of A in the sphere at a distance r from its center at t time. Parameter C_A is the concentration of lemon in the lemon body. The diffusivity of lemon in the lemon body from the center of the lemon to the outside of the lemon up to r_s which is the radius of the lemon is considered for calculation. Then forgetting the mean concentration of A, inside the sphere during t time, we have (total mass transfer in the surface of lemon is based on diffusion):

$$M_A\, A_r N_{Ar}(r = r_s\,, t = t\, *) = \ -4\, M_A\, \pi {r_s}^2 D_{AB}\frac{\partial C_A}{\partial r} \tag{13}$$

By using equations (12 and 13):

$$\frac{\partial C_A}{\partial r}(r = r_s\,, t = t) = -\frac{2}{r_s}\left(C_{A_0} - C_A^*\right)\sum_{n=1}^{\infty} Exp\left(-\frac{Dn^2\pi^2 t}{{r_s}^2}\right) \tag{14}$$

Exiting mass flow from the surface =

$$-4\, M_A\, \pi {r_s}^2 D_{AB} - \frac{2}{r_s}\left(C_{A_0} - C_A^*\right)\sum_{n=1}^{\infty} Exp\left(-\frac{Dn^2\pi^2 t}{{r_s}^2}\right) \tag{15}$$

Then entire mass transfer from the sphere during t time is:

$$N_A' = \int_0^t 4\, M_A\, \pi {r_s}^2 N_{Ar} = M_A\,\frac{8{r_s}^3}{\pi}\left(C_{A_0} - C_A^*\right)\sum_{n=1}^{\infty}\frac{1}{n}\left(1 - Exp\left(-\frac{Dn^2\pi^2\theta}{{r_s}^2}\right)\right) \tag{16}$$

N_A is the rate of mass transfer, but N_A' is the total mass transfer from 0 to t time. It is assumed that the lemon radius is constant because of the low amount of mass transfer from lemon and low concentrates of solute. If assume that the concentration of A in lemon after t time is $\overline{C_A}$, we have;

$$Total\ mass\ transfer = N_A' = \ M_A\left(\frac{4}{3}\,\pi {r_s}^3\right)\left(C_{A_0} - \bar{C}_A\right) \tag{17}$$

From equaling two amounts of N_A:

$$\frac{C_{A_0} - \bar{C}_A}{C_{A_0} - C_A^*} = 1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}Exp\left(-\frac{Dn^2\pi^2 t}{{r_s}^2}\right) \tag{18}$$

$$C_{A_0} - \bar{C}_A : Mass\ transfer\ during\ \theta$$

$$C_{A_0} - C_A^* : Maximum\ possible\ mass\ transfer$$

$$E = \frac{C_{A_0} - \bar{C}_A}{C_{A_0} - C_A^*} = \frac{m_{A_0} - \bar{m}_A}{m_{A_0} - m_A^*}$$

$$E_s = \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}Exp\left(-\frac{Dn^2\pi^2 t}{{r_s}^2}\right)\ ;\ D[=]\frac{m^2}{s}\ , t[=]s\ , r_s[=]m \tag{19}$$

Materials and Methods

Some lemons were obtained from a local market in Tehran (capital of Iran) on a summer day. Five sphere-shaped lemons were selected with 2.75 cm, 3.25 cm, 3.75 cm, 4.25 cm, and 4.75 cm diameters. They were weighted with a digital balance scale. After that, they were stored (hanged) in the same condition in a room with a 25℃ temperature, 30% humidity, and static air (free convection). So, they were weighted at specific times. The amount of $\dot{m}_{A_0} - \bar{m}_A$ was obtained from the data in Table 1 which shows the mass transfer during t time. Also, for a better illustration of this variation, Fig. 1 shows the variation of lemon weights against time. By assuming that %98 of the entire mass transfer occurred over the experiment time, the amount of $\dot{m}_A - \dot{m}_A{}^*$ was obtained at specific times for each lemon and therefore E and E_s were obtained. Then Eq. 19 were solved for diffusivity by MATLAB. So, diffusivity at specific times for each lemon was obtained.

Table 1. Experimental data of lemons with different diameters at specific times in 25℃

Diameter: 2.75 cm		Diameter: 3.25 cm		Diameter: 3.75 cm	
time left(day,hr,sec.)	weight of lemon(g)	time left(day,hr,sec.)	weight of lemon(g)	time left(day,hr,sec.)	weight of lemon(g)
0	12.17	0	20.75	0	31.9
11,46	12.06	11,46	20.56	11,46	31.56
34,16	11.8	34,16	20.09	34,16	30.86
3,11,57	11.05	3,11,57	18.78	3,11,57	28.64
7,9,29	8.95	7,9,29	15.53	7,9,29	22.85
15,10,59	4.99	15,10,59	9.48	15,10,59	14.06
30,10,46	3.06	30,10,46	5.93	30,10,46	8.9

Diameter: 4.25 cm		Diameter: 4.75 cm	
time left(day,hr,sec.)	weight of lemon(g)	time left(day,hr,sec.)	weight of lemon(g)
0	46.79	0	65.74
11,46	45.67	11,46	64.75
34,16	44.52	34,16	62.08
3,11,57	41.89	3,11,57	57.07
7,9,29	34.02	7,9,29	47.05
15,10,59	20.97	15,10,59	27.77
30,10,46	12.26	30,10,46	17.59

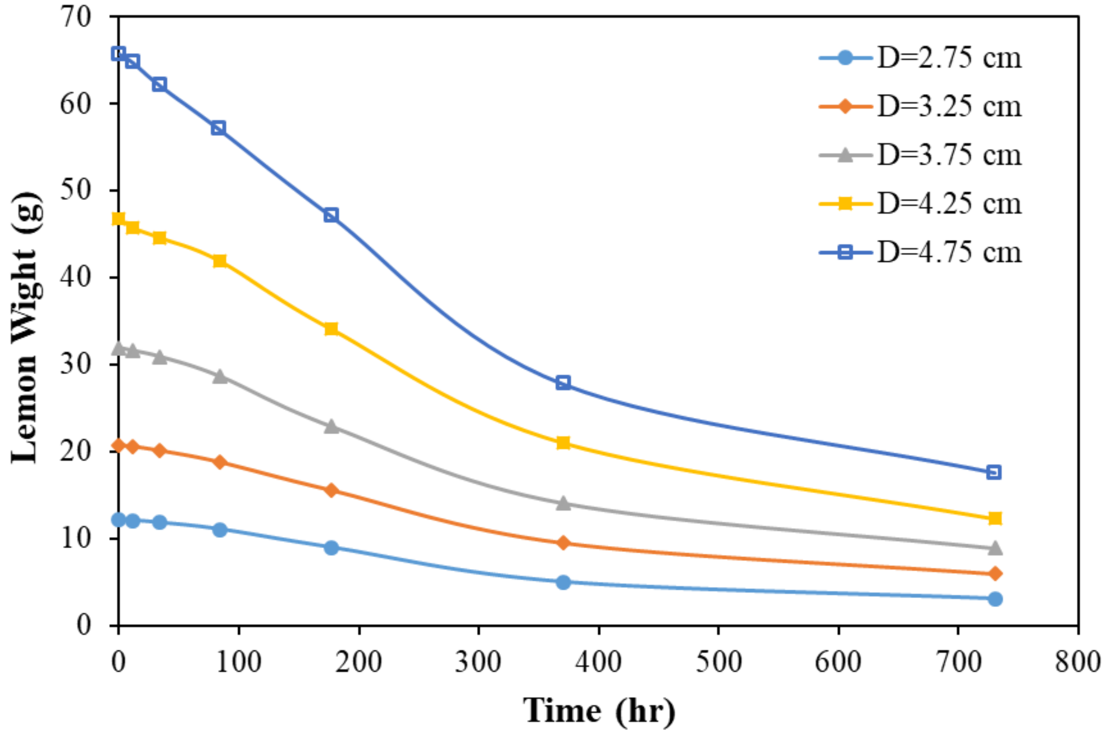

Fig. 1. Experimental data of variation of lemon weights against time

Results and Discussion

It is important to study the kinetics of the drying process and the rate of mass transfer to control this process [4, 18]. It has an important role in knowing about the diffusivity of liquids inside lemon to realize a correct mass transfer mechanism in lemon. The main reason is the dependency between the rate of mass transfer and diffusivity [19]. So, according to the rate of mass transfer, the most efficient way of the drying process can be used. According to the dependency of diffusivity on different factors, various criteria should be studied. Diffusivity is a function of temperature, pressure, humidity, lemon color, time, and lemon diameter. The present experiment was designed the way that all the factors remained constant except time and lemon diameter. Therefore, the dependency from diffusivity on time and lemon diameter was studied. The obtained data of the experiment is shown in Table 1.

According to the data in Table 1, the dependency of diffusivity on time can be achieved, and by using Eq. 19 the amount of diffusivity can be obtained at various times. Therefore, according to Table 1, the dependency of diffusivity on time for various diameters is shown in Fig. 2 to Fig. 6.

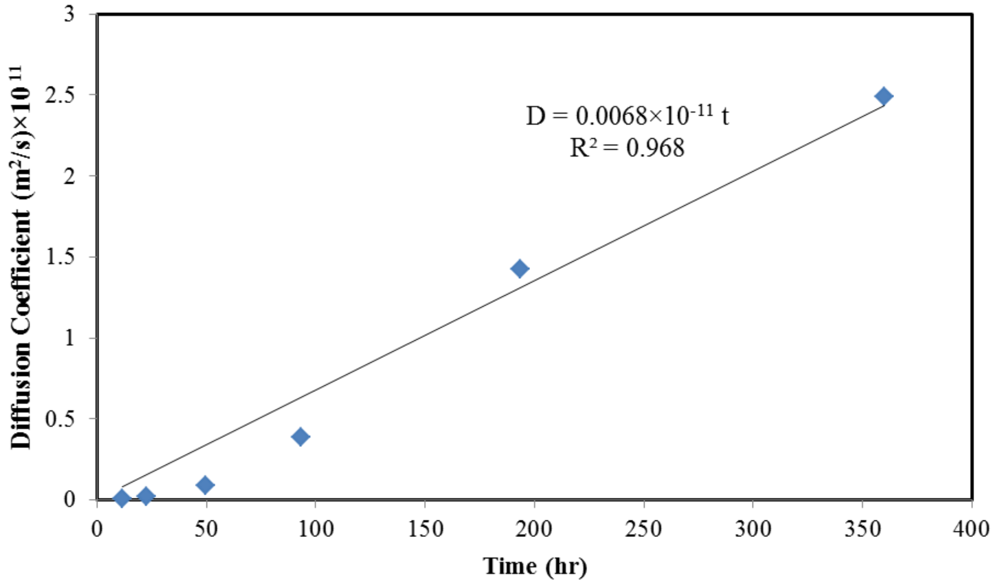

Fig 2. The amount of diffusivity by time for 2.75 diameter lemon

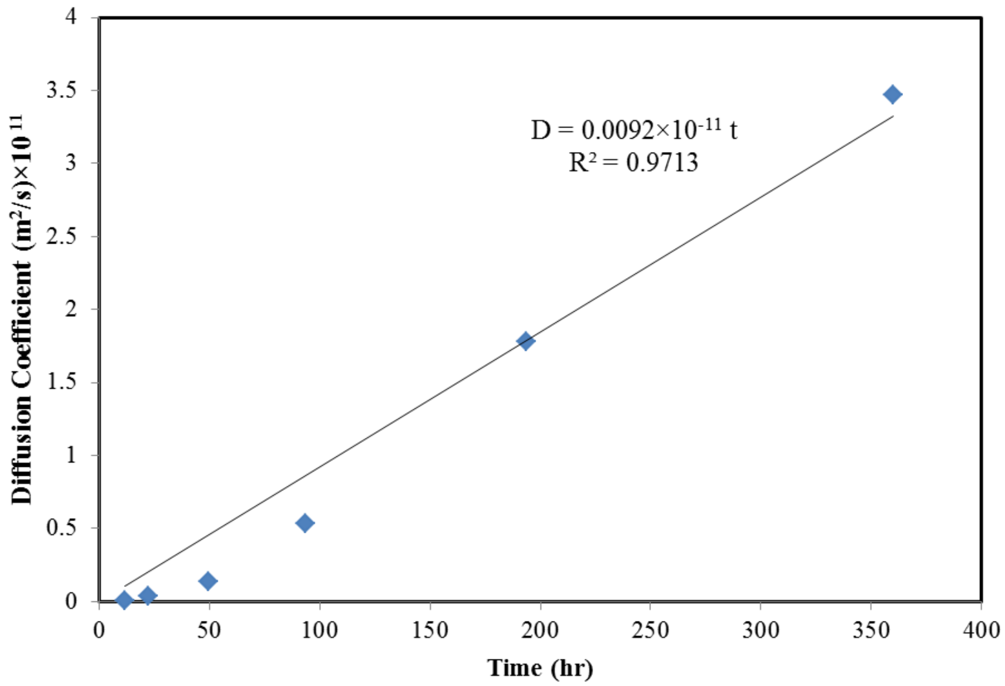

Fig 3. The amount of diffusivity by time for 3.25 diameter lemon

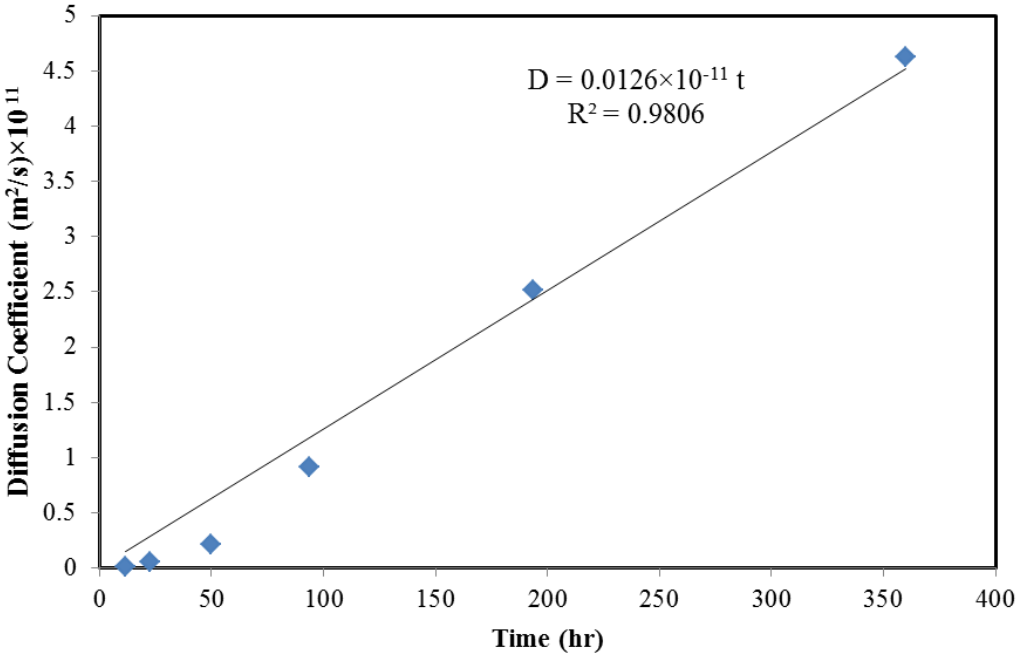

Fig 4. The amount of diffusivity by time for 3.75 diameter lemon

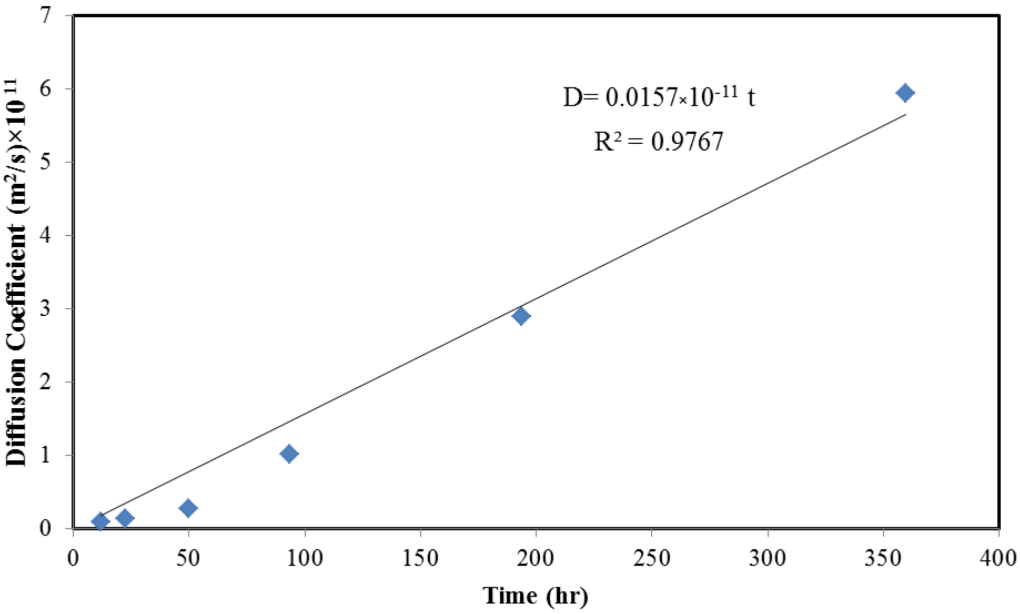

Fig 5. The amount of diffusivity by time for 4.25 diameter lemon

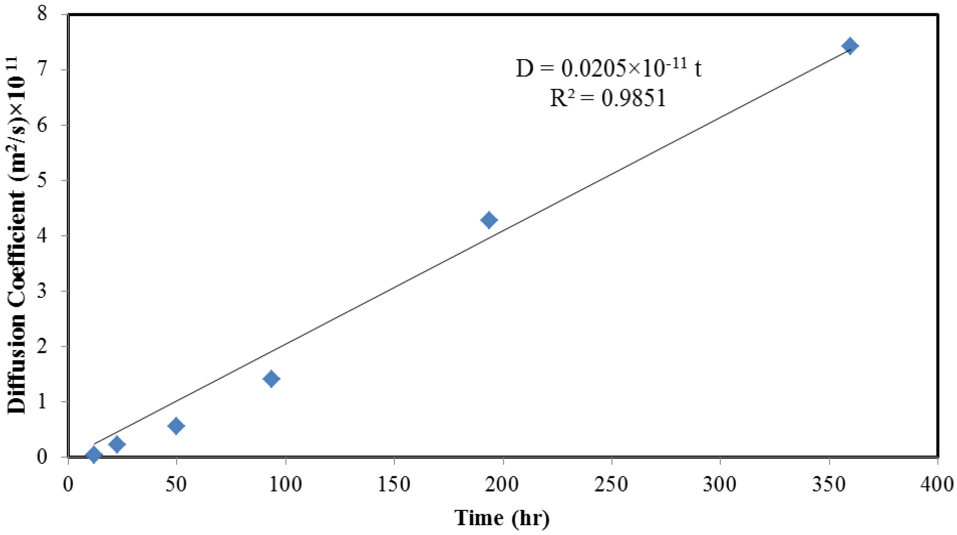

Fig. 6. The amount of diffusivity by time for 4.75 diameter lemon

As shown in Table 1 and Figs. 2 to 6, the linear diffusivity function of time is clear. It is because of increasing pores inside the lemon, that mass transfers in time. Therefore, better conditions for liquid molecules exist to diffuse. As a result, the diffusivity increases, and to investigate the dependency of diffusivity on lemon diameter, an arithmetic mean amount of diffusivity over time for each lemon was chosen. The result is shown in Table 2 and Fig. 7.

Table 2. Diffusivity coefficient in different diameters of lemon

Diameter (cm)	D_{ave} (m^2/s) × 10^{11}
2.750	0.735
3.250	0.994
3.750	1.388
4.250	1.724
4.750	2.323

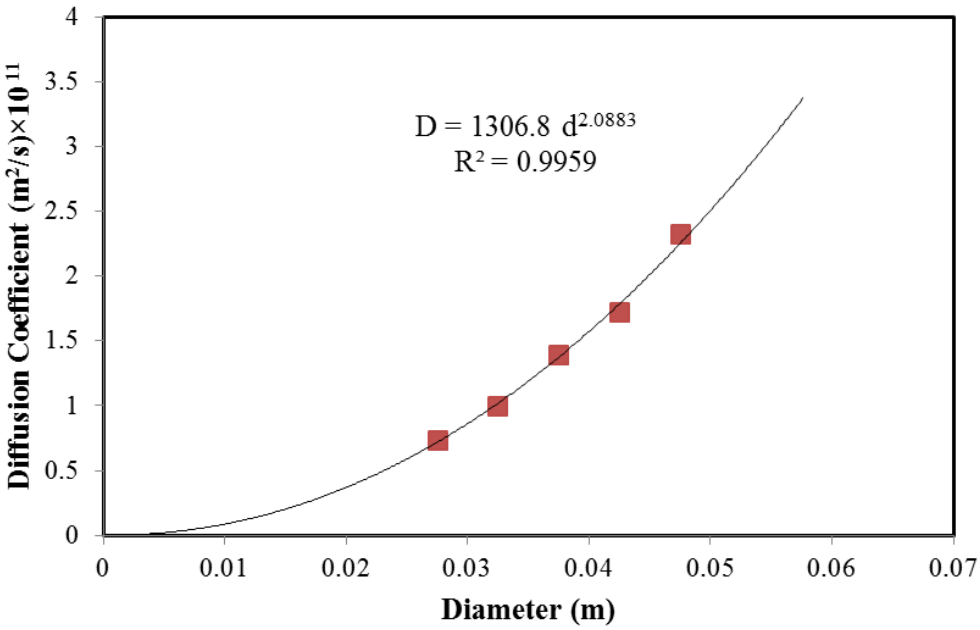

Fig 7. The amount of diffusivity by lemon diameters

According to Fig. 7, the dependency result is achieved using:

$$D = 1306.8\, d^{2.0883} \tag{20}$$

Therefore, the mean value of diffusivity by time for a specific lemon diameter can be obtained. The result shows that the diffusivity of lemon is dependent on the diameter with a power of 2.08. A dimension constant of 1306 is needed for equality of Eq. 20. This means that diffusivity is a function of diameters and a bigger diameter results in greater diffusivity.

Conclusion

In this research, a conceptual study of mass transfer during the drying of lemon has been done in addition to experimental measurements taken for the respective drying conditions. It has included a detailed investigation of the effects of various lemon diameters on the diffusivity of lemon in the drying process in different periods. Then diffusivity coefficient was found theoretically for each situation. In addition, theory-experimental expression was found that shows the relativeness between diffusivity and diameter in lemons. Therefore, diffusivity can be found in other diameters. Furthermore, the time-dependent diffusion distributions for different cases were obtained that would help to expect the time of the drying process and the amount of diffusivity. Diffusion coefficient amounts are about 10^{-11} m^2/s. It shows that assume of mass transfer in solid for lemon is correct. Results show that findings are in considerably high agreement with the experimental data.

Nomenclature

C_A	Concentration of component A
C_{A_0}	Initial value of the concentration of component A
C_A^*	Concentration of component A at lemon center
d	Lemon diameter
D_{AB}	Diffusion coefficient
J	Rate of mass transfer
M_A	Molecular weight
R_A	Reaction rate of component A

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