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Experimental Investigation of Drag Reduction in Turbulent Flow Using Biological and Synthetic Macromolecules: A Comparative Study

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Abstract



It was shown that the concept of drag reducing in the pipe flow with the aid of macromolecules is of great importance in practical engineering applications. In this study, the drag reducing the performance of three biological macromolecules including guar gum (GG), xanthan gum (XG), and carboxymethyl cellulose (CMC) was compared with three synthetic macromolecules including polyethylene oxide (PEO), polyacrylamide (PAM), and polyacrylic acid (PAA). Results showed that all the macromolecules enhanced the DR% except for GG. DR% for almost all of the macromolecules deteriorated with increasing fluid flow rate. On the other hand, DR% enhanced with increasing the pipe diameter for the synthetic polymers but this effect is not obvious for biological polymeric solutions. Maximum DR was 44%, which occur at 1000 ppm concentration of XG at 30 °C and flow rate of 6 l/min and diameter ½ inch. Finally, a new correlation was developed for the prediction of friction coefficient based on the Prandtl-Karman relation with the newly adjusted slope which is a linear function of polymer concentration. This correlation was in excellent agreement with the experimental data.

Keywords: Polymer, Pipe, Friction coefficient, Solution, Practical

1.Introduction

High energy consumption and CO_2 emissions are major concerns in the oil, gas, petrochemical and chemical process industries

+-+. Drag reducing is one of the most effectual methods in this area. The concept of drag reducing of pipe flow with drag reducing agents (DRAs) attracted the engineering applicable usages due to the ability of these agents to decrease pumping power and increment the piping system capacity. Three categories are classified into drag reduction additives: polymers, fibers, and surfactants. Drag reducing polymers (DRPs) are long-chain with ultra-high molecular weight (usually 1 to 10 million).For the first time, Toms [2] reported in the 1940s that by adding long-chain polymers at low concentrations as a few tenths ppm by weight, it is possible to reduce a great decrease in turbulent drag, up to 80%.

Despite many studies in this field, no explanation existed commonly accepted for the drag reducing mechanism [3-7]. Theories explaining the phenomenon of drag reduction are usually classified into two classes in terms of the impacts of the polymer. Joseph et al.[8], De Gennes [9] and Lumley [10]

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proposed some explanations in this regard. The elastic properties of polymers were introduced by Joseph et al.[8] and De Gennes [9] as a reason for drag reduction. Even in a very dilute mode, a polymer solution can be considered a viscoelastic fluid. Actually, as a result of the capability of polymers storing elastic energy, shear waves can be propagated. A normal cut-off fluctuate is provided by these shear waves at great frequencies. The cut off would then overwhelm the small eddies and probably result in a drag reduction. Lumley [10] proposed the mechanism in terms of coiled polymer molecules elongation. The stretching of coiled polymers increases the effective viscosity near the wall and thereby decreases the drag by dampening the thickness of the viscous sub-layer and small vortices.

In experimental investigations on the impacts of polymers on turbulence, decreased performance over time is complicated. Particularly, turbulence-related high shear systems can result in thermal and mechanical degradation of drag-decreasing polymers [11].In these cases, by breaking apart the polymer chains, known as scission, the drag reducing impacts are degraded [12].In many practical applications, determining friction pressure losses of dilute drag decreasing polymer solutions accurately is challenging. Virk et al.[13] demonstrated that the friction factors of definite polymer solutions are much lower compared to the Newtonian fluids. The reason is the polymer solution's viscoelastic properties. In the turbulent flow zone, the viscoelastic fluids' friction factors are quite lower compared to Newtonian or pure viscous fluids. The various polymer solutions' performance was examined by Virk [13] and discovered a trend towards asymptotic maximum drag reduction in all cases.Numerous investigators assessed the different drag affecting parameters. In the following section, drag reducer macromolecules are reviewed separately.

1.1 Polyethylene Oxide (PEO)

Choi et al.[14] studied the impact of the concentration of the very dilute solution of soluble PEO on turbulent DR in a rotating disk flow system. The findings showed that by incrementing the PEO concentration to the maximum DR critical concentration, the DR effectiveness of PEO was increased. DR performance decreased with a further rise in polymer concentration. Kim et al.[15] used PEO as a potential drag reducing in saltwater piping in an ocean thermal energy conversion (OTEC) procedure and assessed the drag reducing properties and PEO mechanical degradation at different concentrations and molecular weights. The results indicated that drag reducing was primarily relied on time and persisted at the limit owing to polymer chains degradation by degrading the polymer chains, drag reducing capacity decreased considerably. The DR efficiency related to temperature was also studied. The findings showed that , however, the percentage of the initial DR was maximum at room temperature where the DR decreased fast. A greater DR effectiveness has been achieved at a lower temperature compared to the higher temperature.

Choi et al.[16] assessed the efficiency of PEO drag reduction in synthetic seawater and indicated that at a concentration of nearly 50 ppm for higher molecular weight PEO, a maximum DR of 30% was achieved. Kim et al. [17] inspected the influences of the molecular weight and concentration of the polyethylene oxides on the drag reducing level through 4 concentrations (1, 5, 10, and, 20 wppm) and 4 molecular weights of the polymer. They showed the maximum drag reducing rate of 50% at a concentration and molecular weight of 20 ppm and 4×10^6 , respectively. The drag reducing impact also tends to increment by increasing the molecular weight and the Reynolds number.

1.2 Polyacrylamide (PAM)

Sung et al.[18] assessed the DR effectiveness of PAM in a rotating disk flow system by comparing it with PEO. The temperature impact on DR was studied for both PAM and PEO at a polymer concentration of 50 wppm. The findings indicated that the temperature increased the mechanical degradation of the PEO chains, while PAM was stable mechanically even at high temperatures. Therefore, for long-lasting transport usages and high temperatures, PAM is an excellent DR additive in the future. The degradation phenomenon was also compared by Sandoval et al. [19] with the use

of a pipe flow device for 3 various aqueous solutions of PEO, PAM and Xanthan gum(XG). PEO and PAM possess flexible chains; however, the XG has a rigid chain structure. The results showed that PAM was as effective as PEO, while the PAM solution's DR efficiency was reduced to a lesser amount compared to PEO. In addition, the drag reduction found by the rigid XG chain dropped shortly in the first stage and persisted constantly different from the PEO and PAM instances. Zhang et al.[20] experimentally studied frictional DR and heat transfer in the two-phase flow of air-water with PAM additives and without it in a horizontal circular tube. They indicated a decrease in the heat transfer coefficients by adding PAM from 36.8% to 70.3% and the pressure drop from 31.9% to 54.7% in comparison to lack of the PAM additive. Raei et al.[21] investigated the impact of PAM on heat transfer and pressure drop in a double tube heat exchanger under turbulent flow. It was reported that the DRA had an insignificant effect on the pressure drop; however, a reduction was shown in heat transfer of about 25%. The influences of the addition of PAM to pure water in a compact heat exchanger on pressure drop and heat transfer were experimentally investigated by peyghambarzadeh et al.[22]. Results showed that the pressure drop continuously decreased by adding PAM to water until concentrations of 100 ppm. The pressure drop started increasingly after this optimum concentration. The maximum DR was 14%. In contrast, by increasing PAM concentration, the total heat transfer coefficient was reduced. In this situation, the overall heat transfer coefficient reduced by 28%.

1.3 Polyacrylic acid (PAA)

Kim et al.[23] utilized a complex system of polymer-surfactants (PAA-SDS) as a drag reducing and assessed the impacts of pH and surfactant on the DR efficiency of PAA in an exterior flow utilizing a rotating disk system. The findings showed that compared to the higher pH, the DR efficiency at pH=4 was smaller representing the connection of the turbulent effects of the polymers in water directly to their chains. They indicated that in a turbulent flow, extending the conformation of PAA is more effective in drag reducing in comparison to the compact helical mode. In addition, increasing the SDS concentration (mol/L) intensified the drag reducing the effectiveness of PAA. Also, Kim et al. [24] investigated the conformational variations of the PAA chains under highs shear flow, and discovered a serious reduction in (DR %) by incrementing the rotational speed of time and disk. They also stated that these drag reducing variations are sensitive to external factors like pH, molecular, and polymer weight of PAA. Zhang e tal. [25, 26] investigated the turbulent DR effectiveness of aqueous poly (acrylamide- co-acrylic acid) copolymers within a rotating disk flow system with different molecular parameters and indicated a highest DR of 45% at the concentration of 50 wppm.

1.4 Guar Gum (GG)

Kim et al. [27] investigated GG's DR behavior using a rotating disk device with 3 various molecular weight fractions in water. They tested the efficiency of GG drag reducing and found that GG is an operative aqueous drag reducing agent and is more constant against mechanical chain degradation compared to the synthetic aqueous drag reducing components such as PEO. The result showed that all the GG solutions increased the certain percentage drag decrease within 62-80% of the primary DR efficiency. Deshmukh et al.[28] studied PAM grafting on GG and compared the polymer of the graft with purified GG and commercial GG. They discovered the good resistance of the purified GG and grafted GG to biodegradation and increased efficiency of drag reduction. GG's mechanical degradation is particularly assessed by Hong et al. [29] in a turbulent flow where utilizing ultra sonication 3 various molecular weights of GG were prepared. A rotating disk system was used to calculate the efficiency of the GG drag reduction as a function of time. They presented two various degradation models of a single relaxation procedure and examined a stretched-exponential model, and it was found that the stretched-exponential is well fitted to the investigational data. Eshghinejadfard et al. [30] experimentally investigate the change in the pressure drop in a quasi-two-dimensional channel flow utilizing different additives (including GG) considering two concentrations

of 100 ppm and 300 ppm. Contrary to previous findings, GG showed no drag reducing in Reynolds numbers below 21,000. Within the low Reynolds range, an increased pressure drop was found by both concentrations. The highest increment of 9.4% was found for 100 ppm GG solution. At larger Reynolds numbers (Re>22,000), an obvious drag reducing was observed.

1.5 Xanthan Gum (XG)

Sohn et al. [31] assessed the impact of different molecular factors on the DR of XG such as polymer concentration, molecular weight, solution's ionic strength, disk rotation speed, and temperature. The results showed that a close relation between the DR efficiency of XG and different molecular parameters. Its greater shear stability in salt solutions and water was documented in comparison with other flexible polymers. In the study of Hong et al. [32], the effectiveness of DR caused by various concentrations of XG was investigated in aqueous KCl solutions in a closed chamber through a rotating disk and it was indicated that mechanical degradation decreased with increased KCl concentration as a function of time. The interaction is allowed by anionic charges on XG allow between the added salt ions for inducing an XG conformational variation in solution leading to the variations in the shear viscosity. The DR improved at higher XG concentrations and the DR efficiency of XG/KCl decreased by increasing KCl concentration because XG's polymeric chain conformation tends to be more rigid, resulting in lower sensitivity to elevated shear conditions.

The drag reducing the effectiveness of PEO, PAM, and XG was analyzed by Andrade et al. [33] through dissolving these 3 polymers with synthetic sea salt and without it in deionized water. Utilizing a double-gap Couette-type rheometer tool, the impact of the salt concentration was investigated on the DR over time. By the existence of salt, the highest DR efficiency for both XG and PEO macromolecular solutions was reduced over time, though, not significant change was found in the DR for PAM solutions. The sharp reduction in effectiveness is related to the structural change from helical to coil by adding salt.

To study the feasibility of enhancing the shear resistance of the hydrolyzed form of polyacrylamide (HPAM), Habibpour et al. [34] prepared various PAM/XG and PAM/GG mixtures and single GG and XG polymer solutions and measured the drag reducing in a closed flow loop. They indicated DR efficiency of both GG and XG solutions directly proportional to polymer concentration and superior mechanical resistance was found in both solutions in turbulent conditions. In dilute HPAM/XG solutions (C<300 wppm), the degree of DR was decreased by adding XG and only shear stability was slightly enhanced. However, in concentrated HPAM/XG solutions (C>450 wppm), with the XG concentration of over the critical overlap concentration; both stability and DR efficiency were significantly enhanced by adding XG.

1.6 Carboxymethyl cellulose (CMC)

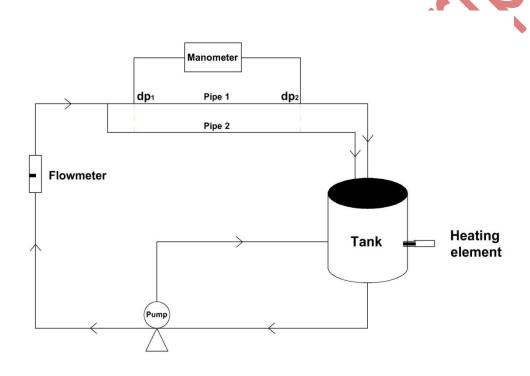
Deshmukhet al. [35] presented a technique to synthesize CMC-based graft copolymers through grafting acrylamide chains onto the CMC backbone while measuring their DR efficacy, biodegradability, and shear stability. Results exhibited that the DR efficacy and decent mechanical shear stability were improved by existing the grafted PAM chains, and it was also found that these factors were reliant on the length and number of the grafts. Peyghambarzadeh et al. [36] investigated the impacts of CMC in laminar flow using two types of CMC (CMC-Hi molecular weight and CMC-Medium molecular weight) at different temperatures and concentrations and measured the pressure drop and the heat transfer coefficient in a air-fined heat exchanger. Their findings indicated that the drag reducing percentage is increased by incrementing temperature, DRA concentration, and fluid flow rate. It was also reported that by incrementing DRA concentration, the overall heat transfer coefficient was continuously decreased. In the study of Biswal and Singh [37], 6 various CMC-g-PAM copolymers were synthesized by changing the quantity of catalyst and monomer, and significant viscosifying and flocculation properties were found by these grafted copolymers.

In the present work, adding PAM, CMC, PEO, PAA, GG, and XG as DRAs into the water during turbulent flow, inside circular smooth tubes was investigated. The impacts of various factors on the pressure drop have been examined. The investigated factors in this research have included been polymer type, polymer concentration, flow rate, temperature, and diameter of the tubes. To have a comprehensive analysis of the results, the full factorial experiment was conducted. Given the number of factors and levels, and, since all the tests have been repeated at least three times, the total number of tests carried out in this study was over 1000.

2. Measurements and materials

2.1 Experimental setup

A schematic view of the experimental apparatus and a realistic photograph are shown in Fig. 1 and 2, respectively.



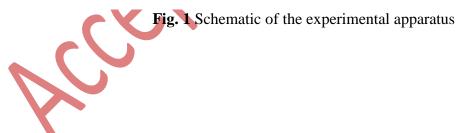




Fig. 2 Photograph of the experimental apparatus

Three concentrations of 6 kinds of dilute polymeric solutions as a drag reducer were examined at three temperatures and flow rates in the device connected with two pipes, each with a various diameter. Pipe No. 1 with a diameter of 0.0127 m is a rough pipe of carbon steel; pipe No. 2 is made of carbon steel with a diameter of 0.01905 m. All pipes are 3 meters long. For recycling the extra liquid to the tank, another pipe is also utilized. The tank is utilized as a storing tank for the solution. The test loop includes one reservoir tank, digital thermostat controller with PID controller, heater, centrifugal pumps, metal valves for closing and opening the flow paths, flow meters, U-shaped manometer, and control box.

The DRA solution is in a cylindrical 20 L carbon steel reservoir tank. At the bottom of the tank, an electric heater is mounted with 6 kW of power that can heat the fluid to a boiling point. A thermostat is joined to this heater to control the temperature with a digital display (BR6FDMP4 models with a precision of ± 0.1 °C) that shows and controls the temperature of the tank. By obtaining the needed temperature, via a centrifugal pump (DELTA Company with a maximum capacity of 50 L/min, 0.75 hp), the solution is pumped into the test section. It is possible to adjust the fluid flow through a valve on the recycle line or a valve installed before the flowmeter. The flow rate was measured using Rotameter (Technical Groups Model sp.gr.1.0) with 1.8–18 L/min flow rate. The accuracy of the flow meter is 0.1 L/min and was calibrated by the time considered for a definite volume of discharging fluid. Utilizing a standard manometer with an accuracy of 1 cm–H₂O, the pressure drop was measured across the test section.

2.2 Materials

To investigate the performance as the drag reducing agent, 6 various water-soluble polymers with high molecular weight were utilized. These polymers are commercially accessible copolymers of polyethylene oxide (PEO), polyacrylamide (PAM), xanthan gum (XG), polyacrylic acid (PAA), guar gum (GG), and carboxymethyl cellulose (CMC). The first three polymers are classified as synthetic flexible molecules, however, the last three are regarded as a natural rigid polymer. The properties of drag reducer agent's and the structures of six main water-soluble polymers are summarized and are shown in table 1.

Name	Chemical Structure	Type Polymer	Supplier	Average molecular weight (g/mol)*
Polyethylene oxide (PEO)		Synthetic	Sigma- Aldrich	2×10 ⁶
Polyacrylamide (PAM)	O NH ₂ n	Synthetic	Sigma- Aldrich	5×10 ⁶
Polyacrylic acid (PAA)		Synthetic	Sigma- Aldrich	1.25×10^{6}
Guar Gum (GG)		Biological	Sigma- Aldrich	1.08 ×10 ⁶
Xanthan Gum (XG)		Biological	Sigma- Aldrich	$4.5 imes 10^{6}$
Carboxymethyl cellulose (CMC) *Stated by manufact	$ \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \end{array}\\ \end{array}\\ \end{array}\\ \end{array} \\ \begin{array}{c} \end{array}\\ \end{array}$ \left(\begin{array}{c} \end{array}\\ \end{array} \left(\begin{array}{c} \end{array}\\ \end{array} \left(\begin{array}{c} \end{array}\\ \end{array}) \\ \left(\begin{array}{c} \end{array}\\ \end{array} \left(\begin{array}{c} \end{array}\\ \end{array} \left(\begin{array}{c} \end{array}\\ \end{array} \left(\begin{array}{c} \end{array}\\ \end{array} \left(\begin{array}{c} \end{array} \left(\begin{array}{c} \end{array}\right) \\ \left(\begin{array}{c} \end{array}\\ \end{array} \left(\begin{array}{c} \end{array}\right) \\ \left(\begin{array}{c} \end{array}) \\ \left(\begin{array}{c} \end{array}) \\ \left(\begin{array}{c} \end{array}) \\ \left(\end{array}) \\ \left(\begin{array}{c} \end{array}) \\ \left(\end{array} \\ \left(\end{array}) \\ \left(\end{array}) \\ \left(\end{array}) \\ \left(Biological	Sigma- Aldrich	15×10 ⁶

Table 1. Specifications and structures of the employed polymers

*Stated by manufacture

Table 2 represents the working parameters ranges and the related uncertainties in measuring them. The uncertainty in measuring the variables was calculated based on Moffat [38]. Also, Table 3 shows the information of all the conducted tests.

	1		
Quantity	Unit	Range	Uncertainty
T (Temperature)	°C	30-50	±0.1
Q (Flow rate)	l/min	6-10	±0.1
D (Diameter)	mm	12.7-19.05	±0.05
C (Concentration)	ppm	0-1000	$\pm 1\%$
Re (Reynolds)	-	8300-30000	±2.2%
ΔP (Pressure drop)	Pa	195-2536	4.6%
f (Friction factor)	-	0.02-0.13	5.8%

Table 2. The parameters and their uncertainty in this work.

Polymer	C (ppm)	T (°C)	Q (l/min)	D (in)
PAA	100-200-360	30-40-50	6-8-10	1/2 -3/4
PEO	10-100-200	30-40-50	6-8-10	1/2 -3/4
XG	100-500-1000	30-40-50	6-8-10	1/2 -3/4
CMC	50-100-200	30-40-50	6-8-10	1/2 -3/4
PAM	10-100-200	30-40-50	6-8-10	1/2 -3/4
GG	10-250-500	30-40-50	6-8-10	1/2 -3/4

2.3 Preparation

The polymer powders (PAM, CMC, PEO, and PAA) were weighed utilizing analytical balance, with an accuracy of ± 1 mg (Mettler Toledo XS603s). The polymer solutions were prepared in a separate tank. At first, the polymer powders were dispersed progressively into the deionized water and gradually stirred at 40 rpm to avoid agglutination of the particles on the surface. For maintaining constant rotating speed (rpm), the impeller's speed was controlled with a speed regulator. Each examination was performed after 24h, the time for completing natural diffusion. This process was approved to prevent any polymer degradation before the test initiation.

The GG solution was prepared using a mechanically stirred impeller with an intermediate speed of almost 100 rpm. Using the low impeller speed, the separation of the polymer coils was prevented before utilizing in experimental apparatus under turbulent flow. By adding the GG in the vortex unceasingly at small intervals, the creation of aggregation was avoided in the solution. The solution was stirred for 5 h and then left for hydration during the night.

Essentially, two homogenous and pseudo homogenous approaches exist for performing drag reducing tests utilizing XG [39]. However, through examinations, it was indicated that the pseudo homogenous technique is more effective; therefore, it was utilized in the present study. For obtaining concentrated stock polymer solutions, a suitable weight of polymer powder was dissolved in deionized water. The solution was further gently magnetically agitated for 24 h till complete dissolving. The solution was left unstirred for at least 12 h, so any structure produced over stirring was relaxed and allowed it to equilibrate.

2.4 Data processing

A valid technique to examine the hydraulic impact of adding polymer to turbulent flow is to measure the pressure drop (friction factor) and to compare the measured number against a base state. For a definite flow rate, the Darcy friction factor is determined by [40]:

$$f = \frac{2D}{\rho \overline{u}^2} \Big(\frac{\Delta P}{L} \Big)$$

where, ΔP represents the pressure drop between a pair of pressure manometer located in the test section and, L shows the distance between them. D denotes the diameter, ρ is the solution density, and \bar{u} shows its mean velocity over the test section.

The Reynolds number is determined normally as:

 $Re = \frac{\rho \tilde{u} D}{\mu R}$

Here μ_p shows the viscosity of the solution.

The effectiveness of the drag reducing polymers is determined by the drag reducing percentage (DR%) in a flowing fluid normally stated quantitatively as follows. Hence, it is possible to formulate the corresponding expressions in other dimensional mounts. The percentage of drag reducing (DR%) is explained as the relative difference between the friction factor f_w of the solvent and f_p , that of the polymeric solution, as:

$$DR\% = \left(1 - \frac{f_p}{f_w}\right) \times 100$$

It is also possible to represent the drag reducing based on friction factor and Reynolds number. In a more perceptive representation, the plot of these dimensionless amounts polymer of solution based on Prandtl–Karman (P–K) parameters like $\frac{1}{\sqrt{f}}$ against $Re\sqrt{f}$ indicates that over a certain Reynold

number, the friction factor declines under that for only pure solvent flow. Basically, there is a separate relationship between these parameters in Newtonian fluids for laminar and turbulent regimes, similar to the polymer-solvent presented in Equations. (4) and (5).

$$\frac{1}{\sqrt{f}} = \frac{\text{Re}\sqrt{f}}{16}$$
Laminar flow
(4)
$$\frac{1}{\sqrt{f}} = 4\text{Log}_{10}(\text{Re}\sqrt{f}) - 0.4$$
Turbulent flow
(5)

 $\overline{\sqrt{f}} = 4 \log_{10} (\text{Re}\sqrt{f}) - 0.4$ Turbulent flow (5) Eq. (6) provides the expression for Virk's MDRA where the friction factor reaches an asymptotic value.

$$\frac{1}{\sqrt{6}} = 19.0 \log_{10}(\text{Re}\sqrt{f}) - 32.4$$

Deionized water was used as a working fluid to examine the friction factor in the turbulent flow regime in order to test the accuracy of the experimental apparatus. The experimental findings were compared with the estimation of the Colebrook equation [40] as Eq. (7):

$$\frac{1}{\frac{1}{2}} = -2 \times \log\left(\frac{2.51}{Ret^2} + \frac{\epsilon/D}{3.7}\right)$$

where ε shows the roughness of the carbon steel tube equals 0.05 mm [40].

2.5. Dynamic rheological measurement

Viscosity is the most imperative property amongst the physical properties of the working fluids with DRA since it is not possible to neglect its variation [3, 22, 36, 41]. In this work, the SVM-3000 Anton Paar viscometer was used to measure the viscosity of various drag reducer fluids. This device measures the kinematic and dynamic viscosity of fluid via only 2.5 ml of the sample.

3. Results& discussion

3.1. Verification with pure water



(1)

(3)

(6)

(7)

First, the accuracy and reliability of the experimental apparatus were checked utilizing distilled water as the working fluid. Through Eq. (1), friction factors (f) at various flow rates were acquired, and the findings were compared with Eq. (7). An acceptable consistency was found between the experimental data and the estimation of the Colebrook equation with an average deviation of 10%. This little deviation can mainly be attributed to the not existence of accuracy in calculating the minor losses caused by abrupt contractions and expansions.

3.2 The viscosity of polymeric fluids

In this work, the viscosity of the solutions including synthetic and biological polymers was experimentally measured at various concentrations and 3 different temperatures of $30 \,^{\circ}$ C, $40 \,^{\circ}$ C, and 50 $^{\circ}$ C, and Fig. 3 represents the results. According to Fig. 3, by increasing the concentration of the DRAs, the polymeric solutions viscosity increments too. However, the slope of these changes is higher for XG, PAA, and GG polymers. Besides, the viscosity of the solutions decreases with increasing temperature.

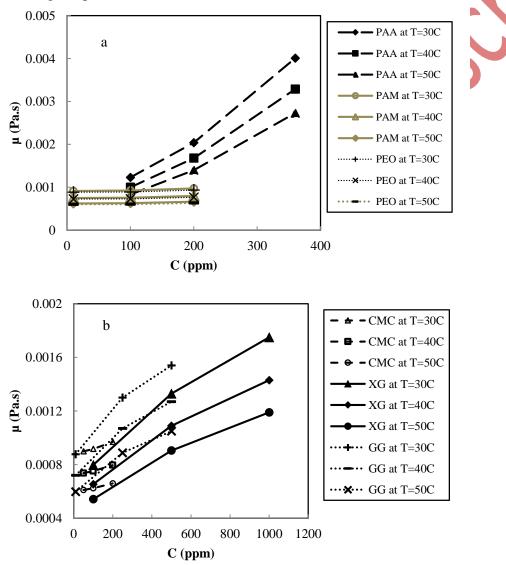
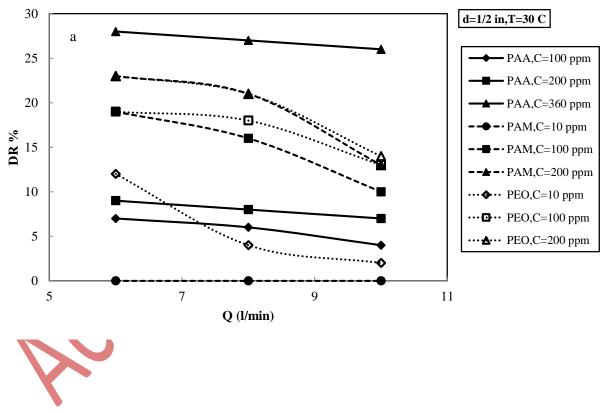


Fig.3. The polymeric fluid viscosity as a function of concentrations and temperatures a) Synthetic DRA b) Biological DRA

3.3. Drag reducing in polymeric solutions

Figs. 4 and 5 (a) and (b) display the variation of DR% as a function of volume flow rate for two types of polymeric solutions including synthetic and biological polymers at the temperature of 30 °C inside the pipes with 1/2 and 3/4 inches diameter. Results show that DR% improves when the polymer concentration increases. This outcome is in complete consensus with the results of Mowla and Naderi [42], and Peyghambarzadeh et al. [36]. But, with increasing GG concentration, the amount of drag has increased. It can probably due to the supererogatory increase of viscosity of the solution with the addition of GG. Habibpour et al.[34] reported that GG has lower efficiency in drag reducing comparing with XG and hydrolyzed PAM due to its low molecular weight and low flexibility. The results of this study also represent that the flow rate (or Re) effect on the drag reducing is not uniform for different polymers. DR% decreases with increasing flow rate for PAM, PEO, and XG while it does not change meaningfully for the cases of CMC and PAA. Indeed, results revealed that increasing flow rate had more effect on PAM and PEO. Since these polymers are flexible, it is probably expected that they are more prone to mechanical degradation due to flow rate increment. Actually, the drag reducing mechanism in flexible polymers is very diverse compared to the rigid ones and it is associated with the microstructure of molecules, based on Virk et al. report [43]. The contradictory results reported by White and Mungal [44], and Peyghambarzadeh et al. [36] reported that DR% increases with increasing fluid flow rate.



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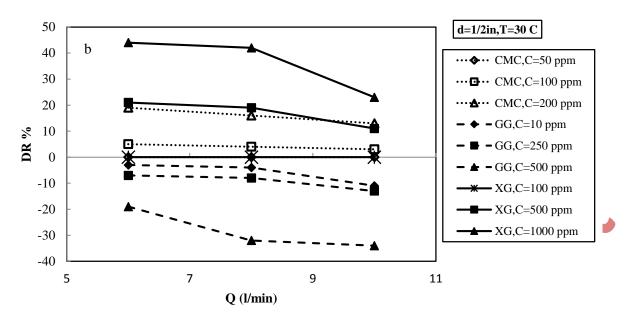
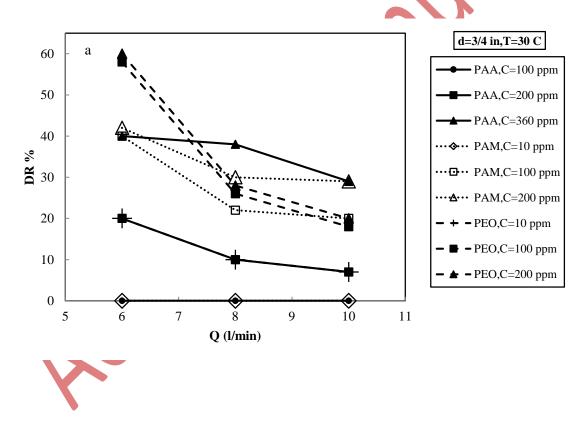


Fig.4 The variation of the DR% with volume flow rate at different concentrations at d=1/2 in a) Synthetic DRA b) Biological DRA



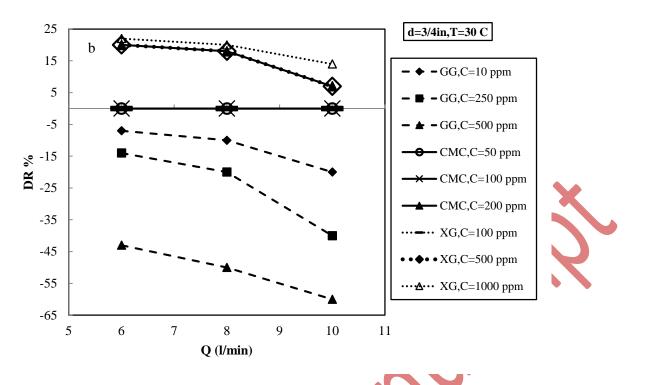


Fig.5 The variation of the DR% with volume flow rate at various concentrations at d=3/4 in a) Synthetic DRA b) Biological DRA

The important roles of temperature contain the impact on polymer solubility in the fluid, polymer degradation, and liquid viscosity. To assess the temperature impact, considering these influences simultaneously is essential. With increasing temperature, the polymer molecules' tendency to extending will increment. The higher temperature will result in more solubility of the DRAs in water flow. The interactions within fluid and DRA involve two problems; one is polymer particle agglomeration during cold flow probably leading to a reduction in polymer solubility in the fluid. Thermal degradation is another problem that may impose damaging impacts on the polymer chai and thereby the DRA performance. Therefore, the impacts of temperature on drag reducing phenomenon strongly rely on the utilized experimental temperature range. Fig. 6 (a) and (b) represent the change in DR% with the polymer concentration and operating temperature at a constant flow rate of 6 l/min inside the ¹/₂ inches pipe. Results show that the effect of temperature on the DR% is not uniform for different polymeric solutions. As can be seen, increasing temperature enhances the DR% for PEO. It can be caused by the lower viscosity of the working fluid at higher temperatures. Furthermore, higher temperatures increase the DRA solubility in distilled water [45]. Probably, these two factors simultaneously increase the influence of PEO at higher temperatures. The mentioned result is in accordance with [4, 22, 36]. On the other hand, DR% for PAA and XG decreases with increasing temperature. In this regard, it was shown that the effect of temperature on the DR% for PAM and CMC is not considerable. Previously, Interthal and Wilski [46] showed that increasing temperature from 5 to 35 °C did not change the DR%. They used partially hydrolyzed PAM at 30 ppm concentration inside the 14 mm diameter pipe and the Re= 100'000. Nevertheless, it is not possible to determine whether a similar temperature trend is true for all researchers since the published studies are very limited in this regard. It can be understood that owing to the complexity of the 6 stated concepts in the DRA performance and flow, providing a firm conclusion is difficult regarding the influence of temperature on DR%.

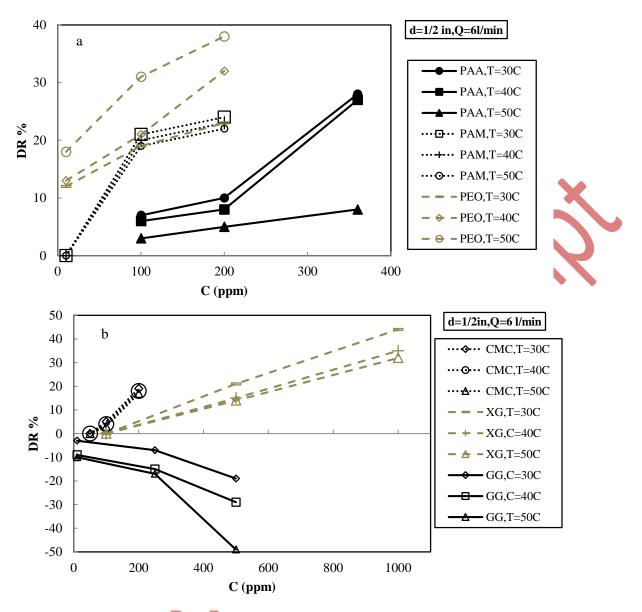


Fig.6 The change in the DR% with concentration at various temperatures (d=1/2 in, Q=6 l/min), a) Synthetic DRA b) Biological DRA

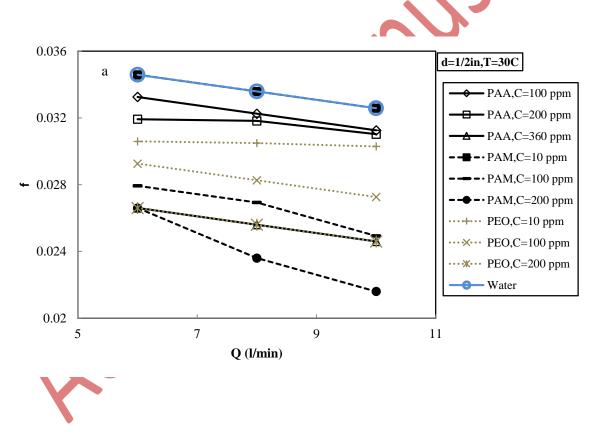
Table 4 shows the average values of DR% for different polymers at their best concentration (which is the highest concentration) inside the $\frac{1}{2}$ and $\frac{3}{4}$ inches pipes. As can be seen, all the polymers enhanced the DR% except for GG that enhances the drag in the fluid flow with average values of 35% and 61% for $\frac{1}{2}$ and $\frac{3}{4}$ inches pipes, respectively. Also, results show that DR% enhances with increasing the pipe diameter for the synthetic polymers of PAA, PAM, and PEO, but this effect is not obvious for biological polymeric solutions like CMC and XG.

Though various researchers investigated the influence of pipe diameter on the efficiency of DRP, the effect of variation in the internal diameter (ID) of the conduit or pipe on the quantity of drag reducing (DR%) is still controversial since the conclusion of researchers are not consistent on this matter. Nevertheless, one of the most complete findings on the effect of DRPs was provided by Interthal and Wilski [46] via factors like pipe diameter. They reported the increase in drag reducing from 66% at 3-mm ID to a peak of 80% at 14-mm ID and then the reduction to 76% at the maximum 30-mm ID. This outcome approved the nonexistence of persistence in the change in drag reducing with pipe diameters. In another similar work conducted by Karami and Mowla [4], the drag reducing of 3

different various solutions was investigated with a similar concentration of 200 ppm and at 29 °C in two rough galvanized iron pipes with 0.0254 and 0.0127 m IDs. They found that the DR was reduced by increasing the diameter of the pipe for all the polymer solutions.

Table 4. The effectiveness of the polymeric DRAs					
DRA	C(ppm)	d(in)	%DR _{ave}	d(in)	%DR _{ave}
PEO	200	1/2	22	3⁄4	31
PAM	200	1/2	18	3⁄4	36
PAA	360	1/2	17	3⁄4	26
CMC	200	1/2	16	3⁄4	14
XG	1000	1/2	29	3⁄4	21
GG	500	1/2	-35	3⁄4	-61

Fig. 7 (a) and (b) demonstrates the variation of Darcy friction factor against volume flow rate at different concentrations for synthetic and biological polymers at 30 °C inside ½ and ¾ inches pipes. Results show that except for the case of GG, distilled water has the greatest friction factor. Furthermore, more decrease in the friction factor could be observed at higher concentrations of the polymeric solution.



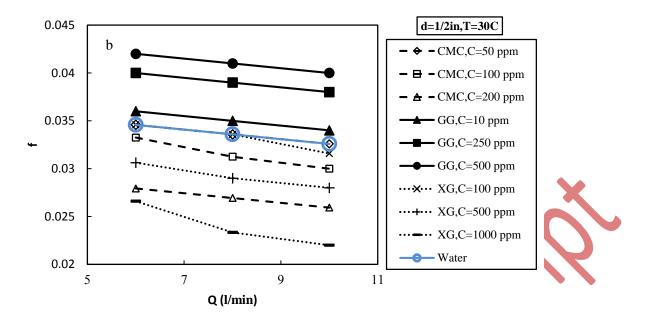


Fig.7 The variation of the friction factor with volume flow rate at various concentrations at d=1/2 in a) Synthetic DRA b) Biological DRA

3.4. Drag reducing in Prandtl–Karman coordinates

Utilizing Prandtl–Karman coordinates, it is possible to compare the degree of DR of polymer solutions regarding the drag reducing boundaries; the start of drag reducing as the departure point from Prandtl–Karman law and maximum drag reducing (MDR) or Virk's asymptote [13, 44].Fig. 8 demonstrates Prandtl-Karman coordinates for different polymers at all the concentrations and constant temperature of 30 °C inside the ½ inches pipe. As can be seen, all the experimental data alight among the Virk and Prandtl-Karman asymptotes. Distilled water data showed about a 12% deviation from the Prandtl-Karman relation which is a confirmation for calibration and accuracy of the apparatus. The experimental friction factor also showed the same deviation with the Colebrook equation.

Results show that for the concentration range of this study, DR is lower than the maximum DR predicted by Virk. So, it is expected that DR% enhances with increasing polymer concentration. PAA and PEO polymeric solutions were approaching Virk maximum drag reducing relation. It is shown that these polymers have higher efficiency in drag reducing. The experimental data for GG had the most distance from Prandtl-Karman prediction which indicates this polymer did not have drag reducing property. Also, Results show that all the data could be predicted with Prandtl–Karman equation with a deviation between -12 to 35 percent.

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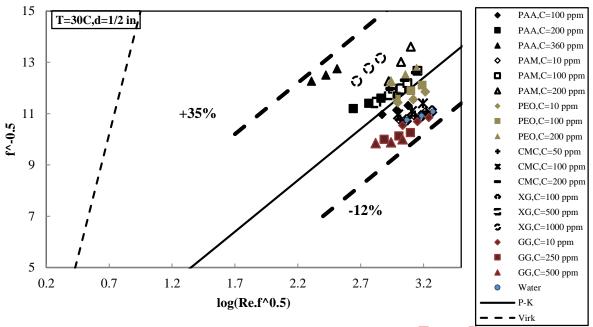


Fig.8 Prandtl-Karman coordinates for polymeric DRA

It should be emphasized that contrary to some other researches [47, 48] that reported drag reducing polymers could reach the Virk relation, the results of the present study indicated that the experimental data have a large deviation from Virk relation. The experimental data is well-aligned with the Prandtl-Karman relation. Therefore, it was tried to forecast the polymers drag reducing at the concentration range of this study using a modified Prandtl-Karman relation. Results discovered that incrementing the polymer concentration enhances the slope of the line in Prandtl-Karman relation. For example, Fig. 9 demonstrates these lines for PAA according to the Prandtl-Karman type relation. It is shown that enhancing the PAA concentration from 100 to 360 ppm increases the slope from 1.852 to 2.848. According to the literature [49], it was proposed that the slope increase regarding the Prandtl-Karman law is proportionate to the square root of polymer concentration (\sqrt{C}) with aproportionality constant that is the representative of the polymer. In this study, strong proportionality obtained for all the polymers to C itself rather than \sqrt{C} . Table 5 indicates the experimental data curve fitting for all the polymers in the turbulent flow regime at 30 °C inside $\frac{1}{2}$ inches pipe. So, the values of A as the slope of the Prandtl-Karman law for each polymer were obtained and reported in Table 5. Meanwhile, the modified presentations of Prandtl-Karman law for each polymer are also shown in Table 5.

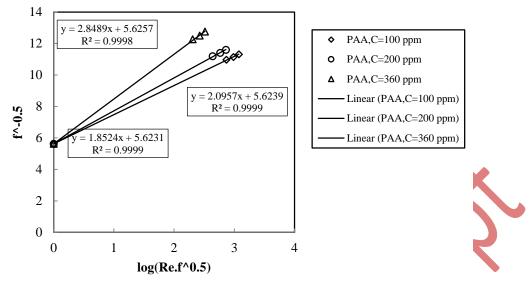


Fig. 9 Linear fit results for PAA (as an example for modification of Prandtl–Karman law)

DRA	Linear fit equation	A	\mathbb{R}^2	Range of Conc.(ppm)
РАА	$\frac{1}{\sqrt{f}} = A \log_{10}(Re.\sqrt{f}) + 5.62$	0.003C+1.404	0.98	[100-360]
РАМ	$\frac{1}{\sqrt{f}} = ALog_{10}(Re.\sqrt{f}) - 0.8$	0.004C+3.754	0.97	[10-200]
PEO	$\frac{1}{\sqrt{f}} = ALog_{10}(Re,\sqrt{f}) + 5.56$	0.001+1.91	0.97	[10-200]
СМС	$\frac{1}{\sqrt{f}} = ALog_{10}(Re.\sqrt{f}) + 4.5$	0.003C+1.848	0.98	[50-200]
XG	$\frac{1}{\sqrt{f}} = ALog_{10}(Re.\sqrt{f}) + 3.14$	0.001C+2.348	0.99	[100-1000]
GG	$\frac{1}{\sqrt{f}} = ALog_{10}(Re.\sqrt{f}) + 6.8$	-0.0004C+1.222	0.94	[10-500]

 Table 5 Modification of Prandtl–Karman equation for polymeric DRA (T=30 °C,d=1/2 in)

Fig. 10 compares the prediction of obtained correlations with the experimental data in Prandtl–Karman coordinates. The new correlations are in excellent agreement with the empirical data. In this graph, the correlation developed by Habibpour et al.[47] was also presented. Habibpour et al. [47] suggested relation for the prediction of friction factor for hydrolyzed PAM at the concentration of 100 and 200 ppm, respectively as follows:

$$\frac{1}{\sqrt{f}} = 6 \log_{10} Re \cdot \sqrt{f} - 4.1$$
(8)
$$\frac{1}{\sqrt{f}} = 9.3 \log_{10} Re \cdot \sqrt{f} - 7.3$$
(9)

It is clear from Fig. 10 that Eq. (8) for the concentration of 100 ppm is in better agreement with the empirical data obtained in this study while the prediction of Eq. (9) for 200 ppm is not acceptable. This may be due to the difference in polymer molecular weight, the extent of hydrolyzing, and operating conditions.

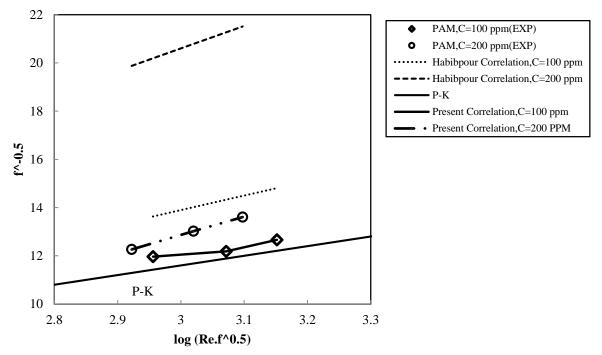


Fig.10 Comparison of the experimental values with the values obtained from the proposed correlation of Habibpour et al.[47] and the present study

4. Conclusion

In this study, a large number of experimental data was gathered for drag reducing properties of almost all water-soluble polymers including PAA, PAM, CMC, GG, XG, and PEO for fluid flow in turbulent regime inside the smooth circular pipes. Furthermore, the effects of different parameters like polymer concentration, temperature, fluid flow rate, pipe diameter, and type of polymer were rigorously studied. Due to a large number of operating factors and their levels of variation, and considering the repeatability tests, more than 1000 experimental data were obtained and analyzed.

The significant findings can be summarized as follows:

- The addition of all the polymers except for GG enhanced the DR%.
- GG deteriorated the drag reducing up to 61%. It did not have drag reducing property.
- **DR%** increases with increasing polymer concentration.
- Change of pipe diameter showed different effects on the drag reducing for different polymers. The efficiency of drag reducing improved for some of them like PEO, PAM, and PAA in larger diameter pipe. This effect was not considered for biological polymers like CMC and XG.
- The influence of temperature on the efficiency of drag reducing was not uniform for all the polymers. Increasing the temperature enhances DR% for PEO while it reduces DR% for PAA and XG. Indeed, the effect of temperature on PAM and CMC was not obvious.
- The experimental data had a large deviation with Virk relation. They scattered in the vicinity of the Prandtl-Karman relation with a deviation of less than 35%.
- A modified version of the Prandtl-Karman relation was proposed for each polymer which could predict the experimental data with better accuracy.

- Considering the probability of polymer degradation according to fluid flow shear stress, it is suggested that the drag reducing properties of the polymers will be studied in the future considering the degradation tendency of the polymers.

Nomenclature

C: Polymer concentration, (ppm) CMC:Carboxymethyl cellulose DR: Drag reduction DRAs:Drag reducing agents DRPs:Drag reducing polymers Exp:Experimental f: Friction factor GG:GuarGum l: Litter L:TubeLength, (m) P: Pressure, (Pa) PID: Proportional-integral-derivative ppm: Parts per million PAA:Polyacrylic acid PAM:Polyacrylamide PEO:Polyethylene oxide OTEC: Ocean thermal energy conversion Q: Volume flow rate, (l/min) Re: Reynolds number SDS: Sodium dodecyl sulfate T: Temperature, (°C) \overline{u} : Velocity, (m/s) XG:XanthanGum

Greek symbols

ε: Roughness, (m)
μ: Viscosity, (Pa.s)
ρ: Density, (kg/m³)
δ: Slope

Subscripts p: Polymeric solution w: Water (solvent)

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